

1S3 (Timoney) Tutorial sheet 4

[Tutorials December 4–8, 2006, January 8–12, 2007]

Name:

Student ID:

1. Show that the hyperbolic functions \cosh and \sinh satisfy the formula

$$\sinh^2 x = \frac{1}{2}(\cosh 2x - 1)$$

(which is an analogue of the trig formula $\sin^2 \theta = (1/2)(1 - \cos 2\theta)$ — with different signs).

2. Show that the equation $x + \cosh x = 2$ has a solution (in \mathbb{R}). [Hint: Intermediate Value Theorem.]
3. Show that if $f(x)$ is differentiable on \mathbb{R} and $f(x) = 0$ has 3 different solutions, then $f'(x) = 0$ must have at least 2 different solutions. [Hint: Rolle's theorem.]
4. Where is the graph $y = \frac{x-1}{(x-2)^2}$ increasing and where is it decreasing?
5. You are planning to make an open rectangular box from an 8 by 15 cm piece of cardboard by cutting squares from the corners and folding up the sides. What are the dimensions of the box with largest volume you can make this way?

Please hand in your work at your first tutorial 1S3 in January. (The tutorial groups may change in January.)

Richard M. Timoney