## **1S3 (Timoney) Tutorial sheet 3** [Tutorials November 20– December 1, 2006]

1. Show that the parametric equations

$$\begin{aligned} x &= 4 + 7\cos t, \\ y &= -2 + 7\sin t \qquad (0 \le t \le 2\pi) \end{aligned}$$

describe points on the circle of radius 7 centered at (4, -2).

Solution: The equation of the circle centered at  $(x_0, y_0)$  and radius r is  $(x - x_0)^2 + (y - y_0)^2 = r^2$ . So the circle we should look at has equation  $(x - 4)^2 + (y + 2)^2 = 49$ . Substituting  $x = 4 + 7 \cos t$  and  $y = -2 + 7 \sin t$  into this the left side of equation we get

$$(x-4)^2 + (y+2)^2 = (7\cos t)^2 + (7\sin t)^2 = 49\cos^2 t + 49\sin^2 t = 49(\cos^2 t + \sin^2 t)$$

and this is always = 49 because  $\cos^2 t + \sin^2 t = 1$  always.

So we have verified that all the points on the parametric curve do lie on the circle (satisfy the equation of the circle).

2. Fill in the following Mathematica command so as to plot the parametric curve in the first question.

ParametricPlot[ {4 + 7 Cos[t], -2 + 7 Sin[t] }, {t, 0, 2 Pi}]

3. Find the equation of the tangent line to the graph  $y = (x + 1)/(x^2 + 1)$  at the point on the graph where x = 2.

Solution: We know the graph of the tangent line to a graph y = f(x) at the point where x = a is given by y = f(a) + f'(a)(x - a) and then what we have to do is to work this out for a = 2 and  $f(x) = (x + 1)/(x^2 + 1)$ .

First

$$f'(x) = \frac{dy}{dx} = \frac{1(x^2+1) - (x+1)(2x)}{(x^2+1)^2} = \frac{x^2+1-2x^2-2x}{(x^2+1)^2} = \frac{-x^2-2x+1}{(x^2+1)^2}$$

and so  $f'(2) = (-4 - 4 + 1)/5^2 = -7/25$ . (This number is the slope of the tangent line). Next f(2) = (2+1)/5 = 3/5 and so y = f(2) + f'(2)(x-2) becomes

$$y = \frac{3}{5} - \frac{7}{25}(x-2).$$

This is the desired equation of the tangent line.

4. For the same graph find the points where the tangent line is horizontal (slope 0).

Solution: We seek points x where f'(x) = 0. We already have f'(x) above and it will be zero when

$$\frac{-x^2 - 2x + 1}{(x^2 + 1)^2} = 0$$
  
$$-x^2 - 2x + 1 = 0$$
  
$$x^2 + 2x - 1 = 0$$
  
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
  
$$= \frac{-2 \pm \sqrt{4 + 4}}{2} = -1 \pm \sqrt{2}$$

Thus the points in question are the points on the graph where  $x = -1 + \sqrt{2}$  and where  $x = -1 - \sqrt{2}$ . (We could perhaps work out the y-coordinates of these points.)

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