1S3 (Timoney) Tutorial sheet 2
[November 7 – 18, 2005]

Name: Solutions

1. Convert $(1010011001)_2$ to octal and $(3146)_8$ to binary using the "3 binary for 1 octal digit" rule.

   Solution:
   $$(1010011001)_2 = (001\ 010\ 011\ 001)_2 = (1231)_8$$
   $$(3146)_8 = (011\ \ 001\ \ 100\ \ 110)_2 = (11001100110)_2$$

2. Convert $(2ab8)_{16}$ to binary using the "4 binary for one hex" rule.

   Solution:
   $$(2ab4)_{16} = (0010\ 1010\ 1011\ 0100)_2 = (10101010110100)_2$$

3. If a computer used 36 bits to store each floating point number, with 26 bits for the mantissa and 10 for the exponent, what would be the largest number it could store and roughly how large is that in decimal?

   Solution: Largest mantissa is (+)$(1.11\ldots1)_2$ with 25 1's (hence $= 2 - \frac{1}{2^{24}} \cong 2$) and the largest exponent is $2^9 - 1 = 511$. That makes the largest possible number almost $2 \times 2^{511} = 2^{512} = 2^2 \times 2^{510} = 4 \times (2^{10})^{51} \cong 4 \times (10^3)^{51} = 4 \times 10^{153}$.

4. Convert $\frac{23}{7}$ to binary.

   Solution: First $\frac{23}{7} = 3 + \frac{2}{7}$ and $3 = (11)_2$. We concentrate on the fractional part $\frac{2}{7}$.
   Imagine the binary expansion as

   $$\begin{aligned} \frac{2}{7} &= (0.b_1b_2b_3\ldots)_2 \\ \text{Double} & \\ \frac{4}{7} &= (b_1.b_2b_3b_4\ldots)_2 \\ \text{Integer parts} & \\ & \boxed{b_1 = 0} \\ \text{Fractional parts} & \\ \frac{4}{7} &= (0.b_2b_3b_4\ldots)_2 \end{aligned}$$

$$
\begin{aligned}
&\text{Double} \\
\frac{8}{7} &= (b_2.b_3b_4b_5\ldots)_2 \\
&\text{Integer parts} \\
&\boxed{b_2 = 1} \\
&\text{Fractional parts} \\
\frac{1}{7} &= (0.b_3b_4b_5\ldots)_2 \\
&\text{Double} \\
\frac{2}{7} &= (b_3.b_4b_5b_6\ldots)_2 \\
&\text{Integer parts} \\
&\boxed{b_3 = 0} \\
&\text{Fractional parts} \\
\frac{2}{7} &= (0.b_4b_5b_6\ldots)_2 \\
&= (0.b_1b_2b_3\ldots)_2
\end{aligned}
$$

Thus the pattern repeats, $b_4 = b_1$, $b_5 = b_2$, etc and so $\frac{2}{7} = (0.\overline{010})_2$. The answer is

$$\frac{23}{7} = 3 + \frac{2}{7} = (11.\overline{010})_2$$

Aside:

In (binary) scientific notation, this is

$$(1.1\overline{010})_2 \times 2^1$$

The mantissa is $(1.1\overline{010})_2$ and the exponent is 1.
