## 1S2 (Timoney) Tutorial sheet 9

[January 21 – 25, 2008]

## Name: Solutions

1. For

$$A = \begin{bmatrix} 2 & 1 & 3 & 0 & 0 & 0 \\ 12 & -16 & 0 & -4 & 0 & 0 \\ 3 & -2 & 0 & 0 & 7 & 0 \\ -2 & 8 & 0 & 0 & 0 & -2 \\ 2 & 4 & 0 & 17 & 18 & -2 \\ 9 & 21 & 102 & -22 & 0 & -2 \end{bmatrix}$$

find trace(A) and  $A^t$ .

Solution:

$$\operatorname{trace}(A) = 2 - 16 + 0 + 0 + 18 - 2 = 2$$
$$A^{t} = \begin{bmatrix} 2 & 12 & 3 & -2 & 2 & 9\\ 1 & -16 & -2 & 8 & 4 & 21\\ 3 & 0 & 0 & 0 & 0 & 102\\ 0 & -4 & 0 & 0 & 17 & -22\\ 0 & 0 & 7 & 0 & 18 & 0\\ 0 & 0 & 0 & -2 & -2 & -2 \end{bmatrix}$$

2. For

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -3 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ -8 & 3 & 1 & 0 \end{bmatrix}$$

find  $B^2$ ,  $(B^t)^2$ ,  $B^3$ ,  $(B^t)^3$  and  $B^4$ . [Hint: use the rule about products of transposes.] *Solution:* 

Aside: Since B is a  $4 \times 4$  strictly (lower) triangular matrix, we know B is nilpotent and in fact we know  $B^4 = 0$ . The calculations in the above example follow a pattern that works in general — the successive powers of a strictly triangular matrix will be more and more triangular, in the sense that each successive power of the matrix has an extra line of zeros parallel to the diagonal.

3. For

$$A = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 4 & -2 \\ 8 & 9 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{bmatrix}$$

find det(A) (using cofactor expansion along the first row). Solution:

$$det(A) = 1 \times det \begin{bmatrix} 1 & 4 & -2 \\ 9 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} - 0 + 3 det \begin{bmatrix} 0 & 1 & -2 \\ 8 & 9 & 0 \\ 0 & 2 & 1 \end{bmatrix} + 0$$
  
$$= 1 det \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - 4 det \begin{bmatrix} 9 & 0 \\ 2 & 1 \end{bmatrix} + (-2) det \begin{bmatrix} 9 & 1 \\ 2 & 0 \end{bmatrix}$$
  
$$+ 3 \left( 0 - det \begin{bmatrix} 8 & 0 \\ 0 & 1 \end{bmatrix} + (-2) det \begin{bmatrix} 8 & 9 \\ 0 & 2 \end{bmatrix} \right)$$
  
$$= 1 - 4(9) - 2(-2) + 3((-8) - 2(16))$$
  
$$= -151$$

Please hand in your work at the end of the hour.

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