1S2 (Timoney) Tutorial sheet 8

[January 14 - 18, 2008]

Name: Solutions

1. For

$$D = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

Find D^{-1} , $D^2 = DD$, D^3 (which means D^2D) and D^4 .

Solution: Since matrix mutiplication of diagonal matrices works so easily (multiply the corresponding diagonal entries to get the diagonal entries for the product, which is again diagonal) we can quickly write down the answers

$$D^{-1} = \begin{bmatrix} \frac{1}{3} & 0 & 0 & 0\\ 0 & -\frac{1}{4} & 0 & 0\\ 0 & 0 & \frac{1}{7} & 0\\ 0 & 0 & 0 & -\frac{1}{2} \end{bmatrix}, \quad D^2 = \begin{bmatrix} 9 & 0 & 0 & 0\\ 0 & 16 & 0 & 0\\ 0 & 0 & 49 & 0\\ 0 & 0 & 0 & 4 \end{bmatrix},$$
$$D^3 = \begin{bmatrix} 27 & 0 & 0 & 0\\ 0 & -64 & 0 & 0\\ 0 & 0 & 343 & 0\\ 0 & 0 & 0 & -8 \end{bmatrix}, \quad D^4 = \begin{bmatrix} 81 & 0 & 0 & 0\\ 0 & 256 & 0 & 0\\ 0 & 2401 & 0\\ 0 & 0 & 0 & 16 \end{bmatrix}.$$

2. For

$$U = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 4 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

find U^{-1} .

Solution: We can use the standard method of row reducing $[U|I_4]$ (to get $[I_4|U^{-1}]$).

| 1 | 0 | 3 | 0 | :1 | 0 | 0 | 0 |
|---|---|---|----|-----|---|---|---|
| 0 | 1 | 4 | -2 | : 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | : 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 | : 0 | 0 | 0 | 1 |

Add 2 times row 4 to row 2

| [1] | 0 | 3 | 0 | :1 | 0 | 0 | 0 |
|-----|---|---|---|-----|---|---|---|
| 0 | 1 | 4 | 0 | : 0 | 1 | 0 | 2 |
| 0 | 0 | 1 | 0 | : 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 | : 0 | 0 | 0 | 1 |

Add -4 times row 3 to row 2 and -3 times row 3 to row 1

Thus

3. For

| $U^{-1} =$ | $\left[\begin{array}{c}1\\0\\0\\0\end{array}\right]$ | 0 1 0 0 | _ | ${3 \\ 4 \\ 1 \\ 0 }$ | $\begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ |
|------------|--|---|---|---|--|
| L = | $\begin{bmatrix} 1\\0\\2\\0 \end{bmatrix}$ | $egin{array}{c} 0 \\ 1 \\ 0 \\ 3 \end{array}$ | $egin{array}{c} 0 \\ 0 \\ 1 \\ 0 \end{array}$ | $ \begin{array}{c} 0 \\ 0 \\ 0 \\ 1 \end{array} $ | |

find L^{-1} .

Solution: We can use the standard method again (of row reducing $[L|I_4]$).

| Γ | 1 | 0 | 0 | 0 | :1 | 0 | 0 | 0 |
|---|---|---|---|---|-----|---|---|---|
| | 0 | 1 | 0 | 0 | : 0 | 1 | 0 | 0 |
| | 2 | 0 | 1 | 0 | : 0 | 0 | 1 | 0 |
| L | 0 | 3 | 0 | 1 | : 0 | 0 | 0 | 1 |
| | | | | | | | | |

Subtract 2 times row 1 from row 3

$$\begin{bmatrix} 1 & 0 & 0 & 0 & : & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & : & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & : & -2 & 0 & 1 & 0 \\ 0 & 3 & 0 & 1 & : & 0 & 0 & 0 & 1 \end{bmatrix}$$

Now subtract 3 times row 2 from row 4

$$\begin{bmatrix} 1 & 0 & 0 & 0 & : & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & : & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & : & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & : & 0 & -3 & 0 & 1 \end{bmatrix}$$
$$L^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 0 & -3 & 0 & 1 \end{bmatrix}$$

4. For L, D and U as in the previous questions, find A = LDU and A^{-1} . [Hint: What is the inverse of a product of invertible matrices?]

Solution:

$$DU = \begin{bmatrix} 3 & 0 & 9 & 0 \\ 0 & -4 & -16 & 8 \\ 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$
$$LDU = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 0 & 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 9 & 0 \\ 0 & -4 & -16 & 8 \\ 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 0 & 9 & 0 \\ 0 & -4 & -16 & 8 \\ 6 & 0 & 25 & 0 \\ 0 & -12 & -48 & 22 \end{bmatrix}$$

Since A = LDU and the inverse of a product is the product of the inverses taken in the reverse order, we know $A^{-1} = (LDU)^{-1} = U^{-1}D^{-1}L^{-1}$ and so

$$D^{-1}L^{-1} = \begin{bmatrix} \frac{1}{3} & 0 & 0 & 0 \\ 0 & -\frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{7} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 0 & -3 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1}{3} & 0 & 0 & 0 \\ 0 & -\frac{1}{4} & 0 & 0 \\ -\frac{2}{7} & 0 & \frac{1}{7} & 0 \\ 0 & \frac{3}{2} & 0 & -\frac{1}{2} \end{bmatrix}$$
$$U^{-1}D^{-1}L^{-1} = \begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{3} & 0 & 0 & 0 \\ 0 & -\frac{1}{4} & 0 & 0 \\ -\frac{2}{7} & 0 & \frac{1}{7} & 0 \\ 0 & \frac{3}{2} & 0 & -\frac{1}{2} \end{bmatrix}$$
$$A^{-1} = \begin{bmatrix} \frac{25}{21} & 0 & -\frac{3}{7} & 0 \\ -\frac{2}{7} & 0 & \frac{1}{7} & 0 \\ 0 & \frac{3}{2} & 0 & -\frac{1}{2} \end{bmatrix}$$

Richard M. Timoney