1S2 (Timoney) Tutorial sheet 7
[January 7 – 11, 2008]

Name: Solutions

1. Write the system of linear equations

$$\begin{array}{rcrcrcrcr} 5x_1 & - & 2x_2 & + & x_3 & - & 4x_4 & = & -3 \\ 2x_1 & + & 3x_2 & + & 7x_3 & + & 2x_4 & = & 18 \\ -x_1 & - & 12x_2 & - & 11x_3 & - & 16x_4 & = & -37 \\ x_1 & + & 2x_2 & - & x_3 & - & x_4 & = & -3 \end{array}$$

in the form of a single matrix equation $A\mathbf{x} = \mathbf{b}$ for suitable matrices A, $\mathbf{x}$ and $\mathbf{b}$. (Make it clear which matrices are A, $\mathbf{x}$ and $\mathbf{b}$.)

Solution: If we take

$$A = \left[\begin{array}{rrrr} 5 & -2 & 1 & -4 \\ 2 & 3 & 7 & 2 \\ -1 & -12 & -11 & -16 \\ 1 & 2 & -1 & -1 \end{array}\right], \mathbf{x} = \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array}\right], \mathbf{b} = \left[\begin{array}{r} -3 \\ 18 \\ -37 \\ -3 \end{array}\right],$$

(so that A is the matrix of coefficients of the unknowns (4×4 in this case as we have 4 equations in 4 unknowns), $\mathbf{x}$ the column vector made up of the 4 unknowns, and $\mathbf{b}$ the column vector made up of the 4 constants on the right hand sides of the equations), then the system is the same as

$$A\mathbf{x} = \mathbf{b}$$

2. For

$$A = \left[\begin{array}{rrr} 2 & -1 & 3 \\ 4 & -4 & -2 \\ -4 & 3 & -4 \end{array}\right]$$

find the inverse matrix A^{-1} (using the method of row-reducing $[A|I_3]$ to reduced row-echelon form).

Solution: We use Gauss-Jordan elimination on

$$\left[\begin{array}{rrrcccc} 2 & -1 & 3 & :1 & 0 & 0 \\ 4 & -4 & -2 & :0 & 1 & 0 \\ -4 & 3 & -4 & :0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{rrrcccc} 1 & -\frac{1}{2} & \frac{3}{2} & :\frac{1}{2} & 0 & 0 \\ 4 & -4 & -2 & :0 & 1 & 0 \\ -4 & 3 & -4 & :0 & 0 & 1 \end{array}\right| \text{oldRow1} \times \frac{1}{2}$$

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & \frac{3}{2} & \frac{1}{2} & 0 & 0 \\ 0 & -2 & -8 & -2 & 1 & 0 \\ 0 & 1 & 2 & 2 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \text{oldRow2} - 4 \times \text{oldRow1} \\ \text{oldRow2} + 4 \times \text{oldRow1} \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & \frac{3}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 4 & 1 & -\frac{1}{2} & 0 \\ 0 & 1 & 2 & 2 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \text{oldRow2} \times \left(-\frac{1}{2}\right) \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & \frac{3}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 4 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & -2 & 1 & \frac{1}{2} & 1 \end{array}\right] \begin{array}{l} \\ \\ \text{OldRow3} - \text{OldRow2} \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & \frac{3}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 4 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & -\frac{1}{4} & -\frac{1}{2} \end{array}\right] \begin{array}{l} \\ \\ \text{OldRow3} \times \left(-\frac{1}{2}\right) \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & 0 & \frac{5}{4} & \frac{3}{8} & \frac{3}{4} \\ 0 & 1 & 0 & 3 & \frac{1}{2} & 2 \\ 0 & 0 & 1 & -\frac{1}{2} & -\frac{1}{4} & -\frac{1}{2} \end{array}\right] \begin{array}{l} \text{OldRow1} - \frac{3}{2} \times \text{OldRow3} \\ \text{OldRow2} - 4 \times \text{OldRow3} \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{11}{4} & \frac{5}{8} & \frac{7}{4} \\ 0 & 1 & 0 & 3 & \frac{1}{2} & 2 \\ 0 & 0 & 1 & -\frac{1}{2} & -\frac{1}{4} & -\frac{1}{2} \end{array}\right] \begin{array}{l} \text{OldRow1} + \frac{1}{2} \times \text{OldRow2} \\ \\ \\ \end{array}$$

Thus the inverse matrix is

$$A^{-1} = \left[\begin{array}{ccc} \frac{11}{4} & \frac{5}{8} & \frac{7}{4} \\ 3 & \frac{1}{2} & 2 \\ -\frac{1}{2} & -\frac{1}{4} & -\frac{1}{2} \end{array}\right]$$

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