1S2 (Timoney) Tutorial sheet 6

[November 28 – December 3, 2007]

Name: Solutions

1. Let $\mathbf{x} = (2, -1, 3, 5)$ and $\mathbf{y} = (3, 2, -4, -2)$ (in $\mathbb{R}^4$). Compute

   (a) $10\mathbf{x} - 7\mathbf{y}$

   Solution: $10\mathbf{x} - 7\mathbf{y} = (20, -10, 30, 50) + (-21, -14, 28, 14) = (-1, -24, 58, 64)$

   (b) $\|\mathbf{x}\|$

   Solution: $\|\mathbf{x}\| = \sqrt{2^2 + (-1)^2 + 3^2 + 5^2} = \sqrt{39}$.

   (c) $\mathbf{x} \cdot \mathbf{y}$

   Solution: $\mathbf{x} \cdot \mathbf{y} = (2)(3) + (-1)(2) + (3)(-4) + (5)(-2) = 6 - 2 - 12 - 10 = -18$

   (d) the distance between $\mathbf{x}$ and $\mathbf{y}$.

   Solution: $\sqrt{(2-3)^2 + (-1-2)^2 + (3+4)^2 + (5+2)^2} = \sqrt{1 + 9 + 49 + 49} = \sqrt{108} = 2\sqrt{27} = 6\sqrt{3}$

   (e) the cosine of the angle between $\mathbf{x}$ and $\mathbf{y}$.

   Solution: We know $\mathbf{x} \cdot \mathbf{y} = \|\mathbf{x}\|\|\mathbf{y}\| \cos\theta$ and so we know from earlier that $-18 = \sqrt{39}\|\mathbf{y}\| \cos\theta$. We still need to calculate $\|\mathbf{y}\| = \sqrt{3^2 + 2^2 + (-4)^2 + (-2)^2} = \sqrt{9 + 4 + 16 + 4} = \sqrt{33}$ and then we know

$$\cos\theta = \frac{-18}{\sqrt{39}\sqrt{33}} = \frac{-18}{3\sqrt{11}\sqrt{13}} = \frac{-6}{\sqrt{11}\sqrt{13}}$$

2. Find the equation of the hyperplane in $\mathbb{R}^4$ through $(1, 2, -6, 5)$ perpendicular to $(3, -2, 1, 8)$.

   Solution: The equation looks like $3x_1 - 2x_2 + x_3 + 8x_4 = \text{const}$ and $(1, 2, -6, 5)$ must satisfy the equation. So

$$3(1) - 2(2) - 6 + 8(5) = \text{const}$$

   or $33 = \text{const}$. So the equation of the hyperplane is $3x_1 - 2x_2 + x_3 + 8x_4 = 33$.

3. For

$$a = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 0 & -2 \\ -4 & 3 & 2 \end{bmatrix}, b = \begin{bmatrix} 0 & 2 & 1 \\ 5 & -5 & 2 \\ 7 & -7 & 1 \end{bmatrix}, c = \begin{bmatrix} \frac{1}{2} & 3 \\ 2 & -1 \\ 1 & 0 \end{bmatrix}$$

   compute

   (a) the size of a (3×3) of b (3×3) and of c (3×2)

   (b) the $(3, 2)$ entry of b

   Solution: The $(3, 2)$ entry of b is -7.

(c) $5a + 2b$

Solution:

$$5a + 2b = \begin{bmatrix} 10 & -5 & 15 \\ 20 & 0 & -10 \\ -20 & 15 & 10 \end{bmatrix} + \begin{bmatrix} 0 & 4 & 2 \\ 10 & -10 & 4 \\ 14 & -14 & 2 \end{bmatrix} = \begin{bmatrix} 10 & -1 & 17 \\ 30 & -10 & -6 \\ -6 & 1 & 12 \end{bmatrix}$$

(d) ac

Solution: We need to take, one by one, the (dot) product of each row of a times each column of c

$$ac = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 0 & -2 \\ -4 & 3 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 3 \\ 2 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 7 \\ 0 & 12 \\ 6 & -15 \end{bmatrix}$$

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