1S2 (Timoney) Tutorial sheet 5
[November 21 – 26, 2007]

Name: Solutions

1. Find the equation of the plane through $(1, 2, 5)$ perpendicular to the line

$$\begin{aligned} x &= 3 + 2t \\ y &= 3 - 2t \\ z &= 1. \end{aligned}$$

Solution: A vector parallel to the line is $2\mathbf{i} - 2\mathbf{j} + 0\mathbf{k}$ and this is then normal to the plane. So the plane has equation $2x - 2y = d$ for some constant d.

As $(x, y, z) = (1, 2, 5)$ is on the plane we have to have $2 - 4 = d$ or $d = -2$ and the equation is $2x - 2y = -2$ (or $x - y = -1$).

2. Find parametric equations for the line in space that passes through both points $(1, 2, 3)$ and $(3, -2, 1)$.

Solution: A vector parallel to the line is the difference of the position vectors of the points $(3, -2, 1)$ and $(1, 2, 3)$ on the line. We get then

$$\mathbf{b} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k} - (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) = 2\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$$

So, using the point $(1, 2, 3)$ and the vector parallel we get parametric equations in vector form

$$\mathbf{P} = (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) + t(2\mathbf{i} - 4\mathbf{j} - 2\mathbf{k})$$

and we expand these into

$$\left\{ \begin{aligned} x &= 1 + 2t \\ y &= 2 - 4t \\ z &= 3 - 2t \end{aligned} \right.$$

3. Find parametric equations for the line of intersection of the two planes

$$\begin{array}{rcrcrcl} 2x & - & 3y & + & 4z & = & 1 \\ x & + & 2y & + & 2z & = & 3 \end{array}$$

Solution: We convert the equations to an augmented matrix and use Gauss-Jordan elimination to reduce the matrix to reduced row echelon form as follows.

$$\left[\begin{array}{cccc} 2 & -3 & 4 & :1 \\ 1 & 2 & 2 & :3 \end{array} \right]$$

$$\left[\begin{array}{cccc} 1 & -\frac{3}{2} & 2 & :\frac{1}{2} \\ 1 & 2 & 2 & :3 \end{array} \right] \text{OldRow1} \times \frac{1}{2}$$

$$\left[\begin{array}{cccc} 1 & -\frac{3}{2} & 2 & : \frac{1}{2} \\ 0 & \frac{7}{2} & 0 & : \frac{5}{2} \end{array}\right] \text{OldRow2} - \text{OldRow1}$$

$$\left[\begin{array}{cccc} 1 & -\frac{3}{2} & 2 & : \frac{1}{2} \\ 0 & 1 & 0 & : \frac{5}{7} \end{array}\right] \text{OldRow2} \times \frac{2}{7}$$

$$\left[\begin{array}{cccc} 1 & 0 & 2 & : \frac{11}{7} \\ 0 & 1 & 0 & : \frac{5}{7} \end{array}\right] \text{OldRow1} + \tfrac{3}{2}\text{OldRow2}$$

Now that we have reduced row-echolon form, we write the equations for this out and get

$$\left\{\begin{array}{ccccc} x & & + & 2z & = \frac{11}{7} \\ & y & & & = \frac{5}{7} \end{array}\right.$$

or

$$\left\{\begin{array}{ccl} x & = & \frac{11}{7} - 2z \\ y & = & \frac{5}{7} \\ z & & \text{free} \end{array}\right.$$

Taking $z = t$ as the parameter, we have

$$\left\{\begin{array}{ccl} x & = & \frac{11}{7} - 2t \\ y & = & \frac{5}{7} \\ z & = & t \end{array}\right.$$

4. Find cartesian equations for the line

$$\begin{aligned} x &= 3 + 2t \\ y &= 3 - 2t \\ z &= 1 + 3t. \end{aligned}$$

Solution: Solving each of the 3 equations for t we get

$$t = \frac{x-3}{2}, \quad t = \frac{y-3}{-2}, \quad t = \frac{z-1}{3}$$

and the caretesian equations are

$$\frac{x-3}{2} = \frac{y-3}{-2} = \frac{z-1}{3}$$

Richard M. Timoney