1S2 (Timoney) Tutorial sheet 3

[November 7 – 12, 2007]

Name: Solutions

- 1. For $\mathbf{v} = -3\mathbf{i} + 7\mathbf{j}$ and $\mathbf{w} = 6\mathbf{i} 3\mathbf{j}$, calculate
 - (a) $\|\mathbf{v} + \mathbf{w}\|$ Solution: $\mathbf{v} + \mathbf{w} = (-3\mathbf{i} + 7\mathbf{j}) + (6\mathbf{i} - 3\mathbf{j}) = 3\mathbf{i} + 4\mathbf{j}$. So $\|\mathbf{v} + \mathbf{w}\| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$.
 - (b) The coordinates of the points in the plane with position vectors v and w. (Write down which is which!)
 Solution: v is the poistion vector of the point (-3,7) and w is the position vector of (6, -3).

2.



(a) Show (on the graph) the point P with coordinates (2, 4, 1) and the point Q with coordinates (1, 2, 5)
 Solution: On diagram

- (b) Sketch the position vectors of the two points (P for P and Q for Q).
 Solution: On diagram (as dashed lines from the origin to P and from the origin to Q).
- (c) Calculate the distance from P to Q.Solution: By the distance formula

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} = \sqrt{(1 - 2)^2 + (2 - 4)^2 + (5 - 1)^2}$$

= $\sqrt{1 + 4 + 16} = \sqrt{21}$

is the distance from P to Q.

(d) Calculate $\|\mathbf{Q} - \mathbf{P}\|$.

Solution: As the vector $\mathbf{Q} - \mathbf{P}$ can be represented by an arrow from the point P to the point Q, the length of the vector must be exactly the distance from P to Q. We've just calculated that as $\sqrt{21}$.

3. Find the equation of the points (x, y, z) that are on the sphere of radius 2 and centre (2, 2, 2). Find an answer without square roots. [Hint: the points on the sphere are those with distance from the center exactly equal to the radius.]

Solution: If we write the description of the points on the sphere as a formula, we get

$$\begin{array}{rcl} {\rm distance}((x,y,z),(2,2,2)) &=& 2\\ \sqrt{(x-2)^2+(y-2)^2+(z-2)^2} &=& 2\\ (x-2)^2+(y-2)^2+(z-2)^2 &=& 4 \end{array}$$

We have squared both sides to eliminate the square root.

Note: Normally we could introduce extra solutions to an equation by squaring both sides. The equation t = 4 has just the one solution, but $t^2 = 16$ has two solutions: t = 4 and t = -4. As the distance between points is never negative, we can find the distance if we know its square. So we do not introduce any extra solutions by squaring both sides.

You might remember that the equation of the circle in the plane with centre (x_0, y_0) and radius r is

$$(x - x_0)^2 + (y - y_0)^2 = r^2.$$

The reasoning we just used for one particular sphere can be copied to show that the equation of the sphere in space with centre (x_0, y_0, z_0) and radius r is

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$

You might try to visualise this sphere. Four of the points on it are $(x_0 \pm r, y_0, z_0)$, $(x_0, y_0 \pm r, z_0)$ and $(x_0, y_0, z_0 \pm r)$.

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