1S2 (Timoney) Tutorial sheet 2
[October 30– November 5, 2007]

Name: Solutions

1. Use Gaussian elimination to reduce this matrix to a matrix in row echelon form

$$\left[\begin{array}{cccc:c} 0 & -2 & 0 & 7 & 12 \\ 2 & -10 & 6 & 12 & 28 \\ 2 & -5 & 6 & -5 & -1 \end{array}\right]$$

Solution:

$$\left[\begin{array}{cccc:c} 2 & -10 & 6 & 12 & 28 \\ 0 & -2 & 0 & 7 & 12 \\ 2 & -5 & 6 & -5 & -1 \end{array}\right] \begin{array}{l} \text{OldRow2} \\ \text{OldRow1} \\ \\ \end{array}$$

$$\left[\begin{array}{cccc:c} 1 & -5 & 3 & 6 & 14 \\ 0 & -2 & 0 & 7 & 12 \\ 2 & -5 & 6 & -5 & -1 \end{array}\right] \text{OldRow1} \times (1/2)$$

(Note that we now have 1 in the top left entry.)

$$\left[\begin{array}{cccc:c} 1 & -5 & 3 & 6 & 14 \\ 0 & -2 & 0 & 7 & 12 \\ 0 & 5 & 0 & -17 & -29 \end{array}\right] \text{OldRow3} - 2 \times \text{OldRow1}$$

(Note that we now have zeros below leading 1 in top left. Leaving first row alone from now.)

$$\left[\begin{array}{cccc:c} 1 & -5 & 3 & 6 & 14 \\ 0 & 1 & 0 & -\frac{7}{2} & -6 \\ 0 & 5 & 0 & -17 & -29 \end{array}\right] \text{OldRow2} \times \left(-\tfrac{1}{2}\right)$$

(Now have leading 1 in second row, second column.)

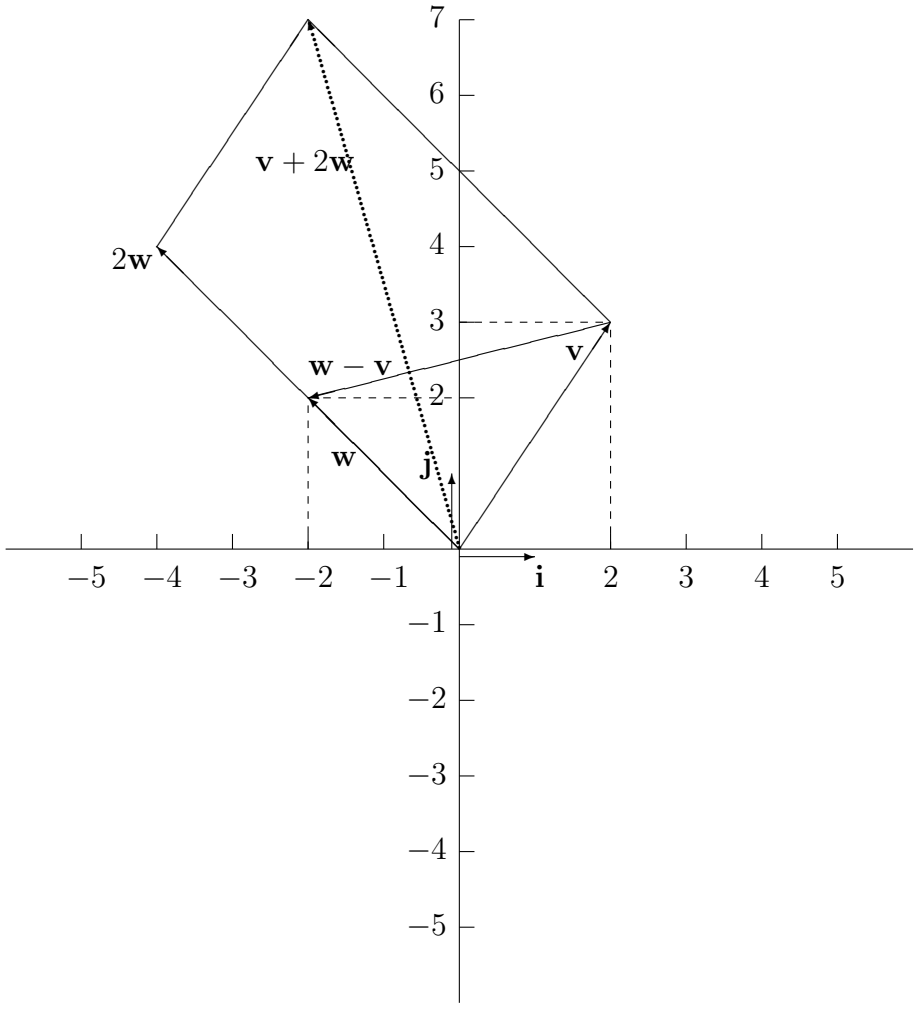
$$\left[\begin{array}{cccc:c} 1 & -5 & 3 & 6 & 14 \\ 0 & 1 & 0 & -\frac{7}{2} & -6 \\ 0 & 0 & 0 & \frac{1}{2} & 1 \end{array}\right] \text{OldRow3} - 5 \times \text{OldRow2}$$

(Now have zeros below second leading 1. Leaving second row alone now, as well as first. Must skip a column now as there is 0 in $(3,3)$ entry now.)

$$\left[\begin{array}{cccc:c} 1 & -5 & 3 & 6 & 14 \\ 0 & 1 & 0 & -\frac{7}{2} & -6 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right] \text{OldRow3} \times 2$$

(Now have leading 1's in all rows, zeros below all. Leading 1 of each later row to the right of those above. So row-echolon form and finished.)

2. Work out graphically the vectors $2\mathbf{w}$, $\mathbf{w} - \mathbf{v}$ and $\mathbf{v} + 2\mathbf{w}$ on the following diagram. Label your diagram clearly. Express all 5 vectors in terms of components.



We can see that $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{w} = (-2)\mathbf{i} + 2\mathbf{j}$. We can compute

$$2\mathbf{w} = 2((-2)\mathbf{i} + 2\mathbf{j}) = (-4)\mathbf{i} + 4\mathbf{j}$$

$$\mathbf{w} - \mathbf{v} = ((-2)\mathbf{i} + 2\mathbf{j}) - (2\mathbf{i} + 3\mathbf{j}) = (-4)\mathbf{i} + (-1)\mathbf{j}$$

and

$$\mathbf{v} + 2\mathbf{w} = (2\mathbf{i} + 3\mathbf{j}) + ((-4)\mathbf{i} + 4\mathbf{j}) = (-2)\mathbf{i} + 7\mathbf{j}$$

Notice that these calculations fit with the diagrammatic results.

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