1S2 (Timoney) Tutorial sheet 10
[January 28 – February 1, 2008]

Name: Solutions

1. Find the area of the triangle in the plane where two sides are of the same length as and parallel to the two vectors $\mathbf{v} = 8\mathbf{i} + 17\mathbf{j}$ and $\mathbf{w} = -3\mathbf{i} + 4\mathbf{j}$.

   Solution: This area of the parallelogram with $\mathbf{v}$ and $\mathbf{w}$ as sides is the absolute value of the determinant where the rows are the components of $\mathbf{v}$ and $\mathbf{w}$:

   $$\det \begin{bmatrix} 8 & 17 \\ -3 & 4 \end{bmatrix} = 8 \times 4 - 17 \times (-3) = 83$$

   As this turns out to be positive the area of the parallelogram is 83. The area of the trinagle is then half that, $83/2$.

2. Find the volume of the parallelopipeed in space where three of the edges are of the same length as and parallel to the vectors $\mathbf{u} = -3\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}$, $\mathbf{v} = 11\mathbf{j} + 14\mathbf{k}$, and $\mathbf{w} = 2\mathbf{k}$.

   Solution: This area is the absolute value of the determinant where the rows are the components of $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$:

   $$\det \begin{bmatrix} -3 & 4 & 8 \\ 0 & 11 & 14 \\ 0 & 0 & 2 \end{bmatrix} = (-3) \times 11 \times 2 = -66$$

   The volume is 66.

3. Use row-reduction to find

   $$\det \begin{bmatrix} 2 & 1 & 3 & 0 & 0 & 0 \\ 12 & 7 & 18 & -4 & 0 & 0 \\ -4 & -2 & -6 & 0 & 7 & 0 \\ -2 & -1 & -3 & 0 & 0 & -2 \\ 6 & 3 & 9 & 17 & 18 & -2 \\ 10 & 5 & 11 & -22 & 0 & -2 \end{bmatrix}$$

   Solution:

   $$\det \begin{bmatrix} 2 & 1 & 3 & 0 & 0 & 0 \\ 12 & 7 & 18 & -4 & 0 & 0 \\ -4 & -2 & -6 & 0 & 7 & 0 \\ -2 & -1 & -3 & 0 & 0 & -2 \\ 6 & 3 & 9 & 17 & 18 & -2 \\ 10 & 5 & 11 & -22 & 0 & -2 \end{bmatrix} = 2 \det \begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} & 0 & 0 & 0 \\ 12 & 7 & 18 & -4 & 0 & 0 \\ -4 & -2 & -6 & 0 & 7 & 0 \\ -2 & -1 & -3 & 0 & 0 & -2 \\ 6 & 3 & 9 & 17 & 18 & -2 \\ 10 & 5 & 11 & -22 & 0 & -2 \end{bmatrix}$$

$$
\begin{aligned}
&= 2\det\begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} & 0 & 0 & 0 \\ 0 & 1 & 18 & -4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 17 & 18 & -2 \\ 0 & 0 & -4 & -22 & 0 & -2 \end{bmatrix} \\
&= -2\det\begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} & 0 & 0 & 0 \\ 0 & 1 & 18 & -4 & 0 & 0 \\ 0 & 0 & -4 & -22 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 17 & 18 & -2 \\ 0 & 0 & 0 & 0 & 7 & 0 \end{bmatrix} \\
&= -2(-4)\det\begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} & 0 & 0 & 0 \\ 0 & 1 & 18 & -4 & 0 & 0 \\ 0 & 0 & 1 & \frac{11}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 17 & 18 & -2 \\ 0 & 0 & 0 & 0 & 7 & 0 \end{bmatrix} \\
&= -8\det\begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} & 0 & 0 & 0 \\ 0 & 1 & 18 & -4 & 0 & 0 \\ 0 & 0 & 1 & \frac{11}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 17 & 18 & -2 \\ 0 & 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 7 & 0 \end{bmatrix} \\
&= -8(17)\det\begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} & 0 & 0 & 0 \\ 0 & 1 & 18 & -4 & 0 & 0 \\ 0 & 0 & 1 & \frac{11}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 1 & \frac{18}{17} & -\frac{2}{17} \\ 0 & 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 7 & 0 \end{bmatrix} \\
&= 136\det\begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} & 0 & 0 & 0 \\ 0 & 1 & 18 & -4 & 0 & 0 \\ 0 & 0 & 1 & \frac{11}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 1 & \frac{18}{17} & -\frac{2}{17} \\ 0 & 0 & 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 \end{bmatrix} \\
&= 136(7)(-2) = -1904
\end{aligned}
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