Mathematics 121 2004–05
Exercises 1
[Due Friday October 29th, 2004.]

1. Prove the following assertions based on the axioms (P1) - (P12) for $\mathbb{R}$:
   (a) $-(-a) = a$ for each $a \in \mathbb{R}$
   (b) $-a = (-1) \cdot a$ for each $a \in \mathbb{R}$
   (c) There is a unique number (one and only one) in $\mathbb{R}$ with the properties for the multiplicative identity 1. [Hint: We know there is such a number by the axiom we assume, but the point is to check that another element $1' \in \mathbb{R}$ with the property $1' \cdot a = a = a \cdot 1'$ for each $a \in \mathbb{R}$ has to be $1' = 1$.]
   (d) Show that each $a \in \mathbb{R}$ with $a \neq 0$ has a unique multiplicative inverse. [Hint: Again we have such an inverse by assumption, but the point is to show that another one is the same. Note: we can then denote it $\frac{1}{a}$ or a^{-1} and we call it the reciprocal of a because we know we are referring to a single number. If numbers could have two different reciprocals we would be in danger of causing confusion by referring to both by the same symbol.]

2. Show that the following cancellation law holds for additive equations

$$a, b, c \in \mathbb{R}, a + c = b + c \Rightarrow a = b$$

   [Hint: The notation $\Rightarrow$ stands for 'implies' and the idea is to show a little theorem or lemma that whenever we are in a situation where the left hand statements are true then it will always be the case that the right hand side is true.]

3. Show that the following cancellation law holds for multiplicative equations

$$a, b, c \in \mathbb{R}, c \neq 0, a \cdot c = b \cdot c \Rightarrow a = b$$

4. Show that $a \cdot 0 = 0$ for each $a \in \mathbb{R}$.

5. Show that if $a, b \in \mathbb{R}$, $a \neq 0$ and $b \neq 0$, then their product $a \cdot b \neq 0$.

6. Show that $3 \neq 0$ (where $3 \in \mathbb{R}$ means $2 + 1$ and 2 means $1 + 1$). [Hint: This is the only place where the order axioms (P10) - (P12) are needed.]

It may take a little while to understand the point here of deducing everything from the small number of rules or axioms (P1) - (P12). We will probably run out of patience before we check everything, but we can in principle verify all the ordinary rules for arithmetic and solving simple equations from these axioms (P1) - (P12).