Michelson Interferometer Lab Report

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Abstract

In this experiment the famous Michelson interferometer and some of its uses are investigated. When set up correctly the Michelson interferometer is used in conjunction with a gas cell and vacuum pump to investigate the dependence of the refractive index of air on the pressure and to simply measure the refractive index of air. It is also used to measure the wavelength difference of a close doublet, the yellow pair in the mercury spectrum. Finally the band pass of a wavelength filter is measured using the interferometer.

1 Theory

Figure 1 shows the Michelson Interferometer. Light from the source passes through the beam splitter which divides the light along two paths. One part is transmitted to mirror M_1, the other is reflected to mirror M_2. These two rays reflect back to the beam splitter where they recombine and proceed toward the eyepiece where interference is observed.

A compensator is placed along the path to M_1 because the ray going to M_2 passes through the beam splitter three times whereas the ray going to M_1 passes through only once. This ensures that should we have a polychromatic light source, such as white light, each wavelength will still have travelled the same distance. If we did not have the compensator present each wavelength would travel a different distance in the beam splitter and this would make the situation more complicated.

A micrometer is provided which allows movement of M_2 and a pair of screws on the back of M_1 allows for its adjustment in two perpendicular directions. If M_1 is adjusted to be parallel to M_2 then the observer viewing from the eyepiece will see the light reflected from M_2 and the image of M_1 as being reflected from a parallel-sided air block of thickness d. This can be thought of as thin-film interference. The path difference t is

$$t = 2d$$

But for an observer looking at angle θ to the normal of M_2 (or M_1) the path difference is actually

$$t = 2d \cos\theta$$

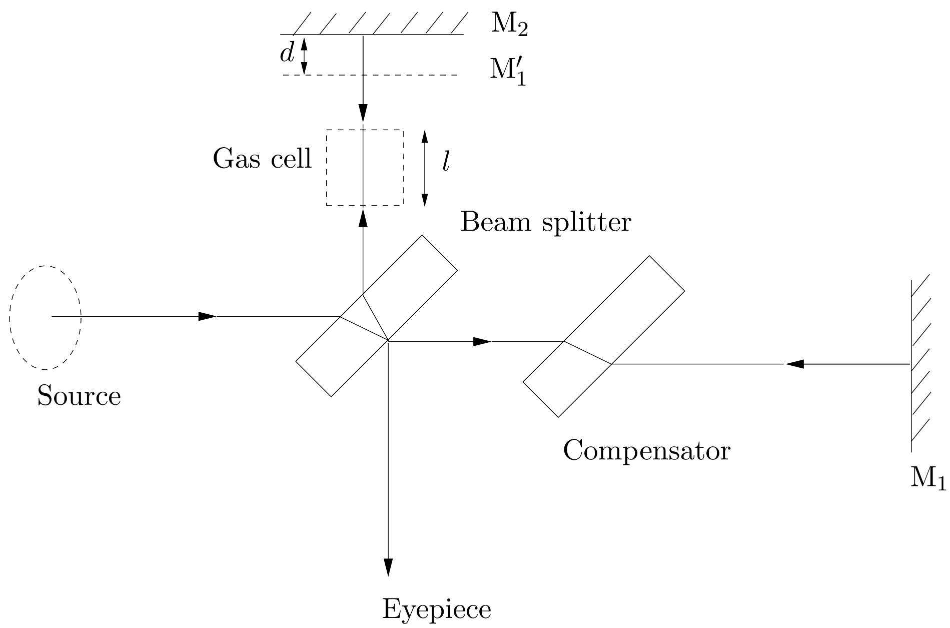

Figure 1: The Michelson Interferometer

Since only the beam going to M_1 is ever reflected on an air-glass interface there is a π phase change between the two beams, therefore the condition for minima is

$$2d\cos\theta = m\lambda$$

This also means that at $d = 0$ the central fringe is dark and not bright.

The position of zero order fringe ($d = 0$) can be found using white light. Looking at the equation above only the central fringe of the white light interference pattern will be dark, the others will be coloured due to the presence of different wavelengths in the white light.

If M_2 is moved to the position of zero order fringe it is superimposed with M_1'. If then M_1 is slightly adjusted so as to create a thin wedge of air between M_1' and M_2, vertical straight parallel lines can be obtained.

1.1 Refractive index of air

To determine the refractive index of air the interference patterns between the gas cell full of air and the gas cell close to a vacuum are compared. If we assume the apparatus is initially set to the position of zero order fringe so $d = 0$ and the gas cell is initially a vacuum then the optical path length of the cell is just $2l$, where l is the length of the cell. When the air is allowed back into the cell the optical path length is $2ln_a$ where n_a is the refractive index of air. Therefore the path difference is

$$t = 2ln_a - 2l$$

so

$$m\lambda = 2l(n_a - 1) \tag{1}$$

where m is the order of the fringe or in the case of the apparatus not initally at the zero order fringe this is the fringe shift.

As the pressure of the air is varied from a vacuum up to atmospheric pressure the density of the gas inside the gas cell increases. The speed of light is slower as the density of a medium is increased so the refractive index of the gas increases with increasing pressure.

1.2 Spectral lines

If there are two closely spaced spectral lines in the spectruum of the light entering the interferometer, for example the mercury yellow pair centered on 578.0nm, the resulting interference pattern will be a superposition of two interference patterns which have almost equal spacings. Figure 2 shows this effect for two patterns of almost equal spacing. As we can see the pattern is an alternating pattern of sharp lines and minima of visibility.

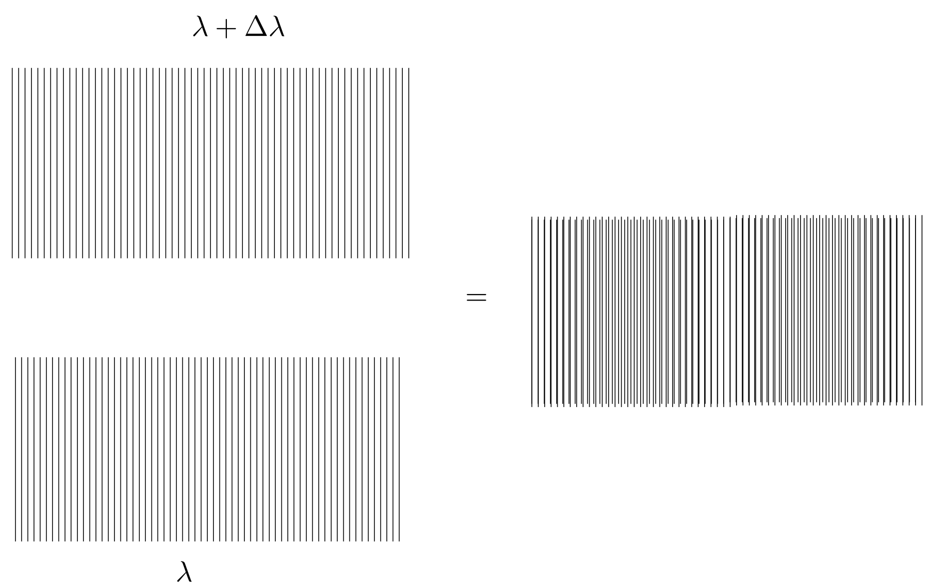

Figure 2: Two interference patterns superimposed.

A minimum of visibility occurs when a bright fringe of one interference pattern lies directly on a dark fringe of the other. This first occurs when

$$m(\lambda + \Delta\lambda) = (m + \frac{1}{2})\lambda$$

where m is the order of the fringe and reoccurs if $B = \frac{3}{2}, \frac{5}{2}, ...$ in the formula

$$m(\lambda + \Delta\lambda) = (m + B)\lambda$$

So moving from one minimum of visibility to the other we have

$$m(\lambda + \Delta\lambda) = (m + 1)\lambda$$

where m is now the fringe shift between the two minima. With a bit of manipulation we have

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{m} \tag{2}$$

If a band of wavelengths are present then the first minimum of visibility will occur at

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{2m} \tag{3}$$

as before but the fringes will not reappear because the wavelengths have ∞ as their LCM. This also means that the wider the band of wavelengths present, the fewer fringes will be present because the order of the fringe at dissapearence will decrease.

2 Experimental Method

2.1 Refractive index of air

2.1.1 Mercury green light

The interferometer was set up for parallel lines with the mercury lamp with the mercury green filter. The gas cell was pumped out. The air was then let back in and the number of fringes passing were noted. The experiment was firstly done with one member of the group looking at the fringes and another controlling the air flow. This proved to be inaccurate and instead one member of the group both controlled the air flow and counted the fringes.

2.1.2 White light

The mercury green light set up was used to calibrate the micrometer. The number of fringes passing in 0.04mm was noted. The interferometer was then set to the zero order fringe with the white light source. The air was pumped out of the gas cell and the micrometer reading was noted. The air was then let slowly back in, adjusting the micrometer to keep the zero order fringe in the center of view at all times. The value of the micrometer with the zero order fringe centered when the gas cell was at atmospheric pressure was noted.

2.2 Dependence of the refractive index of air on pressure

The interferometer was set up with the mercury lamp with the mercury green filter. The gas cell was pumped out completely. The air was let slowly back in by the person counting the fringes, they also counted out the fringes as they passed. The other members of the group took down the fringe count at intervals of 10% of atmospheric pressure.

2.3 Wavelength difference of a close doublet

The interferometer was used with the mercury lamp with the mercury yellow filter. The micrometer readings corresponding to moving from succesive minima were recorded. The same calibration as in section 2.1.2 was used to find m, the fringe shift.

2.4 Band pass of a wavelength filter

The green filter was used with the white light source to give a band of wavelengths. The number of visible fringes was counted.

3 Results

3.1 Refractive index of air

3.1.1 Mercury green light

Fringe shift	
46	standard atmospheric pressure 101.3 kPa
45	ambient temperature: 23 ^{o}C
48	
48.5	

3.1.2 White light

30 fringes -	0.04 mm

Micrometer readings for the zero order fringe centered

Vacuum:	18.75 ± 0.01 mm
Atmospheric pressure:	18.68 ± 0.01 mm

3.2 Dependence of the refractive index of air on pressure

This experiment was done twice so we could have an error for the fringe shift.

% atm	Fringe shift	
	1^{st}	2^{nd}
10	5	4.5
20	10	9.5
30	15	14
40	19	19
50	24	25
60	29	30
70	34	34.5
80	39	39.5
90	44	44.5
100	49	49

3.3 Wavelength difference of a close doublet

Minimum	Micrometer reading (mm)	Error (mm)
1	19.36	0.01
2	19.75	0.01
3	20.15	0.01
4	20.55	0.01
5	20.94	0.01

3.4 Band pass of a wavelength filter

$2m = 24 \pm 1$

4 Discussion

4.1 Measurement of the refractive index of air

The ambient temperature and standard atmospheric pressure were noted because the refractive index of air depends on both. However they were not used in getting the accepted value for $n_a - 1 = 2.92 \times 10^{-4}$.

4.1.1 Mercury green light

Ignoring the first two results since they were obtained using very innacurate means and taking the two final readings from section 3.2 we have

$$m = 48.6 \pm 0.2$$

the error being the standard error in the mean. From equation (1)

$$n_a - 1 = \frac{m\lambda}{2l}$$

$$n_a - 1 = \frac{(48.6 \pm 0.2)(546 \times 10^{-9})}{(2)(50 \times 10^{-3}})$$

$$n_a - 1 = (2.65 \pm 0.01) \times 10^{-4}$$

This is not within experimental error of the expected value of 2.92×10^{-4} for $n_a - 1$. This may have been due to the fact that a complete vacuum was not present in the gas cell in the beginning.

4.1.2 White light

The difference in the two readings of the micrometer is 0.07 ± 0.02 mm. So the equivalent number of mercury green fringes to this shift is

$$m = 30\left(\frac{(0.07 \pm 0.02) \times 10^{-3}}{0.04 \times 10^{-3}}\right)$$

$$m = 52 \pm 14$$

so from equation (1) again

$$n_a - 1 = \frac{(52 \pm 14)(546 \times 10^{-9})}{(2)(50 \times 10^{-3})}$$

$$n_a - 1 = (2.8 \pm 0.8) \times 10^{-4}$$

This is within experimental error of the expected value. However this is also much less accurate than the method involving mercury light. This method is used when much larger path differences have to be determined and so the number of fringes passing cannot be measured directly.

4.2 Dependence of the refractive index of air on pressure

% atm	Fringe shift,m	Δm	$(n_a - 1) \times 10^{-4}$	$\Delta(n_a - 1) \times 10^{-4}$
10	4.75	0.25	0.26	0.01
20	9.75	0.25	0.53	0.01
30	14.5	0.5	0.79	0.03
40	19	0	1.0	0
50	24.5	0.5	1.33	0.03
60	29.5	0.5	1.61	0.03
70	34.25	0.25	1.87	0.01
80	39.25	0.25	2.14	0.01
90	44.25	0.25	2.42	0.01
100	49	0	2.68	0

Figure 3 shows the linear dependence of n_a on the air pressure. The general trend is obvious from the theory but why it is linear I do not know.

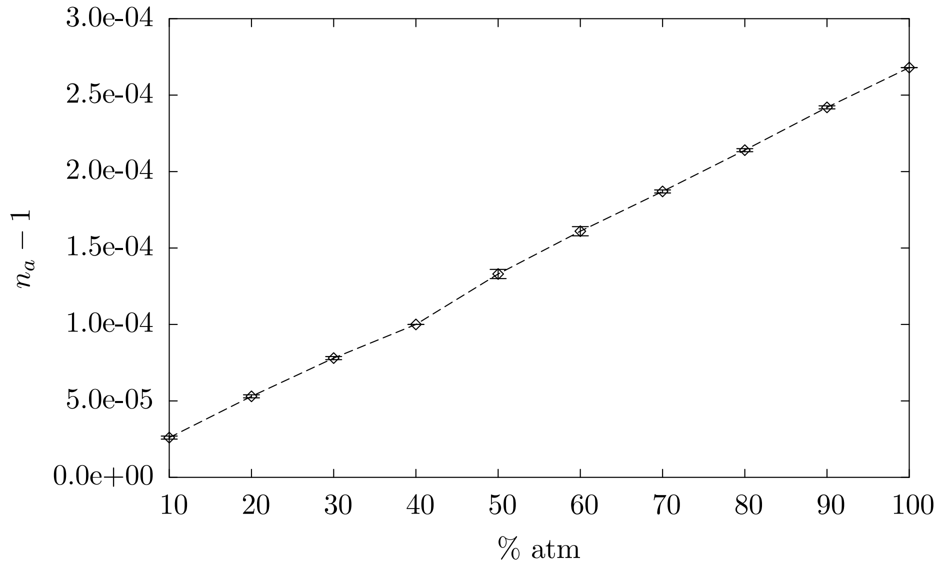

Figure 3: The dependence of the refractive index n_a on pressure is linear, with slope 2.71×10^{-6}.

4.3 Wavelength difference of a close doublet

The average wavelength of the mercury yellow pair in question is 578.0 nm. So λ in equation (2) is 578.0 nm. By taking the difference between the first minimum and succesive minima we can get different values for the difference

Minima	Micrometer difference (mm)	Error (mm)
1–2	0.39	0.02
1–3	0.395	0.01
1–4	0.397	0.01
1–5	0.395	0.005

giving 0.395 ± 0.01 mm for the difference.
Then converting this into a fringe shift

$$m = 30\left(\frac{(0.395 \pm 0.01) \times 10^{-3}}{0.04 \times 10^{-3}}\right)$$

$$m = 296 \pm 7$$

From equation (2) we have

$$\Delta\lambda = \frac{\lambda}{m}$$

$$\Delta\lambda = \frac{578 \times 10^{-9}}{296 \pm 7}$$

$$\Delta\lambda = 1.95 \pm 0.05\text{nm}$$

This is not within experiment error of the expected value of $\Delta\lambda = 2.11$ nm but it is close. This may have been due to the fact that the count of 30 fringes for 0.04 mm may have been wrong, since a small number out (i.e. 1 or 2) means a large difference in $\Delta\lambda$.

4.4 Band pass of a wavelength filter

From equation (3) we have

$$\Delta\lambda = \frac{\lambda}{2m}$$

$$\Delta\lambda = \frac{546 \times 10^{-9}}{24 \pm 1}$$

$$\Delta\lambda = 23 \pm 1\text{nm}$$

so the band pass of the green filter is 23±1 nm.

5 Conclusions

The two obtained values for the refractive index of air were

$$n_a - 1 = (2.65 \pm 0.01) \times 10^{-4}$$

$$n_a - 1 = (2.8 \pm 0.8) \times 10^{-4}$$

The refractive index was found to be a linear function of air pressure with slope 2.71×10^{-6}. Why it is a linear function is not yet clear to me.

The wavelength difference of the mercury yellow doublet was measured to be 1.95 ± 0.05 nm.

Finally the band pass of the given green filter was measured to be 23 ± 1 nm.

References

[1] John Beynon. *Introductory University Optics*. Prentice Hall.

[2] Jurgen R. Meyer-Arendt. *Introduction to Classical and Modern Optics*. Prentice Hall.