Geiger Counter Lab Report

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Abstract

The Geiger counter is used to determine the half-life of a radiactive source, irradiated indium. The dead time of the Geiger tube is also measured using two sources.

1 Apparatus and Theory

1.1 The Geiger Tube

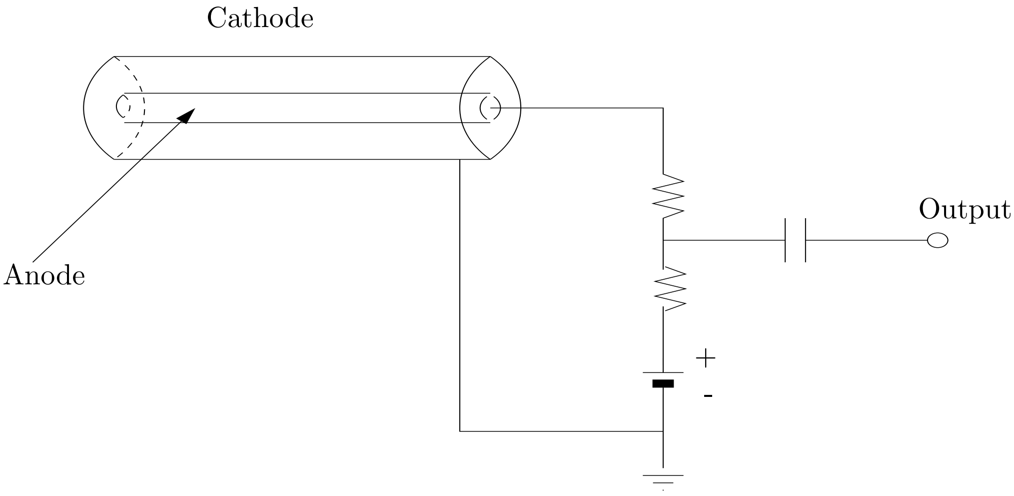

Figure 1: The Geiger Tube

The Geiger tube is a gas-filled radiation detector. As shown in Figure 1 it consists of a cylindrical outer shell (cathode) which is negative with respect to an inner wire (anode). A typical Geiger tube is filled with a mixture of argon and ethanol.

If an alpha particle (say) enters the Geiger tube it will leave a stream of ions and electrons along the path which it travels. These electrons will then be accelerated greatly towards the anode at the center of the tube.

The moving electrons will then ionize other atoms by collisions. This produces a so called avalanche. Photons produced in the avalanche can interact with the cathode where photoelectric emission occurs thus starting another avalanche. This process allows a singe alpha particle to produce an electron cascade which spreads along the length of the Geiger tube, away from the initial avalanche. This is known as a Geiger-Mueller discharge.

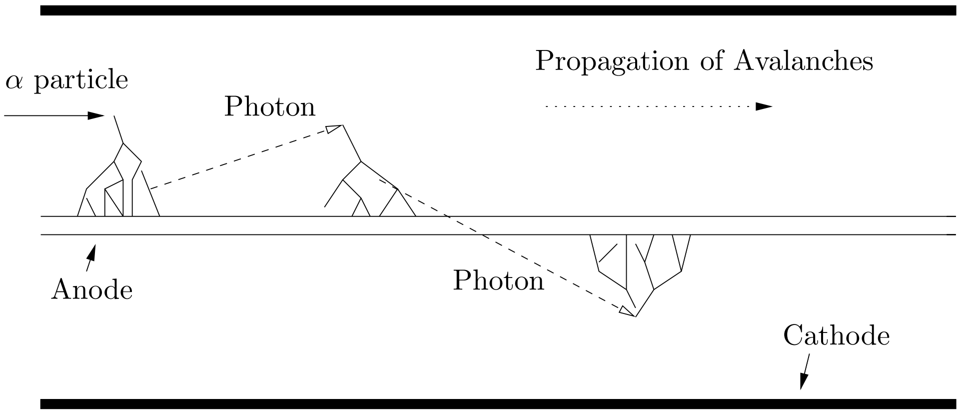

Figure 2: The mechanism of the Geiger tube.

The electrons are much faster moving than the positive ions so the result is a thick layer of positive ions surrounding the anode. This layer builds, reducing the electric field potential between it and the anode, until avalanche production has stopped. This dense sheath moves outwards towards the cathode. When it reaches the cathode the ions could free more electrons thereby starting the process all over again and making the detector useless. To prevent this either a 'quenching circuit' is used or a gas, typically ethanol, is added to the tube. The positive sheath as it moves towards the cathode collides with the ethanol molecules, ionizing it. The ethanol molecule prefers to quickly dissociate than to emmit a photon. This prevents further electron production at the cathode. However this also means that the Geiger tube has a limited lifetime since the ethanol will be used up in the quenching process.

1.2 Dead time

The time for the dense sheath to move out enough so another avalanche can occur is called the 'dead time' of the Geiger tube. If the count rate is recorded for two souces (say m_1 and m_2) and then both together (m), let the true rates be n_1, n_2, and n. Clearly m will be less than the sum $m_1 + m_2$ because of the dead time. If the dead time is τ then the system is 'dead' for a fraction $m\tau$ of the time. So we lose $nm\tau$ of the true counts due to the system being ‘dead’. Since the true count is given by

$$n = m + \text{lostcounts}$$

we have

$$\begin{aligned} n &= m + nm\tau \\ n &= \frac{m}{1 - m\tau} \end{aligned}$$

also the true counts add up

$$n = n_1 + n_2$$

so

$$\begin{aligned} \frac{m}{1 - m\tau} &= \frac{m_1}{1 - m_1\tau} + \frac{m_2}{1 - m_2\tau} \\ \frac{m}{1 - m\tau} &= \frac{m_1 + m_2 - 2m_1m_2\tau}{(1 - m_1\tau)(1 - m_2\tau)} \\ m(1 - (m_1 + m_2)\tau + m_1m_2\tau^2) &= (1 - m\tau)(m_1 + m_2 - 2m_1m_2\tau) \\ m - m(m_1 + m_2)\tau + mm_1m_2\tau^2 &= m_1 + m_2 - 2m_2m_1\tau - m(m_1 + m_2)\tau + 2mm_1m_2\tau^2 \\ mm_1m_2\tau^2 - 2m_1m_2\tau + m_1 + m_2 - m &= 0 \\ \tau^2 - \frac{2\tau}{m} + \frac{m_1 + m_2 - m}{mm_1m_2} &= 0 \end{aligned}$$

This shows we can find the dead time of the Geiger tube if we measure the rate for two sources together and seperately.

If there is a dead time τ' associated with the counting system this may or may not affect the answer. If $\tau' \leq \tau$ then the measured dead time would be the dead time of the Geiger tube. However if $\tau < \tau' \leq 2\tau$ then the measured count would be halved and, from the equation above, the corresponding measured dead time would be doubled. Similarly if $2\tau < \tau' \leq 3\tau$ the count would be divided by three and the dead time would be tripled and so on.

1.3 Half-life

The law of radioactive decay states that the rate of decay is proportional to the number of nuclei in the sample:

$$-\frac{\mathrm{d}N}{\mathrm{d}t} = \lambda N$$

$$\frac{\mathrm{d}N}{N} = -\lambda \mathrm{d}t$$

integrating:

$$N = N_0 \exp(-\lambda t)$$

where N_0 is the number of nuclei present initially in the sample. If we differentiate to find the rate as an funciton of time:

$$R = R_0 \exp(-\lambda t)$$

where $R_0 = -\lambda N_0$, the initial rate. So a graph of $\ln(R)$ vs. t should be a straight line and a graph of R vs. t should be an exponential decay.

The time taken for the rate to decrease by half is called the half-life and so at $t_{1/2}$:

$$\begin{aligned} \frac{R_0}{2} &= R_0 \exp(-\lambda t_{1/2}) \\ t_{1/2} &= \frac{\ln(2)}{\lambda} \end{aligned}$$

1.4 Counting Statistics

The probablility of observing x counts in one observation period can be shown to be a Poisson distribution:

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$

where μ is the average. The standard deviation of a Poisson distribution is

$$\sigma = \mu^{1/2}$$

so the error in a count is taken to be the root of the count.

2 Method and Results

2.1 Half-life of radioactive indium

The counter unit and the amplifier and bias supply unit were set up according to instructions given. The counter was run for 2 mins to get a rough idea of the background rate, the count was 72 ± 8. The error was thought to be too high so the counter was run for 10 mins and the count was 641 ± 25 so the background rate was found to be $R_b = .53 \pm .02$ counts per second.

The indium source was then placed in the Geiger tube and one minute counts were taken every five minutes for an hour. The results are:

Count	Δ Count	R	R_c	ΔR	t(mins)
794	28	13.2	12.7	.5	0
753	27	12.6	12.1	.5	5
679	26	11.3	10.8	.4	10
642	25	10.7	10.2	.4	15
538	23	9.0	8.5	.4	25
547	23	9.1	8.6	.4	30
509	23	8.5	8.0	.4	35
446	21	7.4	6.9	.4	40
401	20	6.7	6.2	.3	45
440	21	7.3	6.8	.3	50
360	19	6.0	5.5	.3	55
368	19	6.1	5.6	.3	60

where R is the rate in counts per second, R_c is the corrected rate ($R - R_b$) and ΔR is the error in the rate. The errors being given by

$$\begin{aligned} \Delta\text{Count} &= \sqrt{\text{Count}} \\ \Delta R &= \frac{\Delta\text{Count}}{\text{Count}} R \\ &= \frac{R}{\sqrt{\text{Count}}} \end{aligned}$$

2.2 Dead time of Geiger tube

The count for 2 mins for the two carbon-14 sources were measured seperately and together. This was a sufficient amount of time because the counts were so high. The background radiation was not accounted for because it was much less than the error in the rate.

	Count	Δ Count	R	ΔR
m_1	65131	255	543	2
m_2	48333	220	403	2
m	94251	307	785	3

3 Analysis and Discussion

3.1 Half-life of radioactive indium

Fig 3 shows the graphs of Rate vs. time and ln(Rate) vs. time with exponential and linear fits respectively. The error in $\ln(R)$ being given by

$$\Delta \ln(R) = \frac{\Delta R}{R}$$

These graphs agree well with theory since these fits were expected.

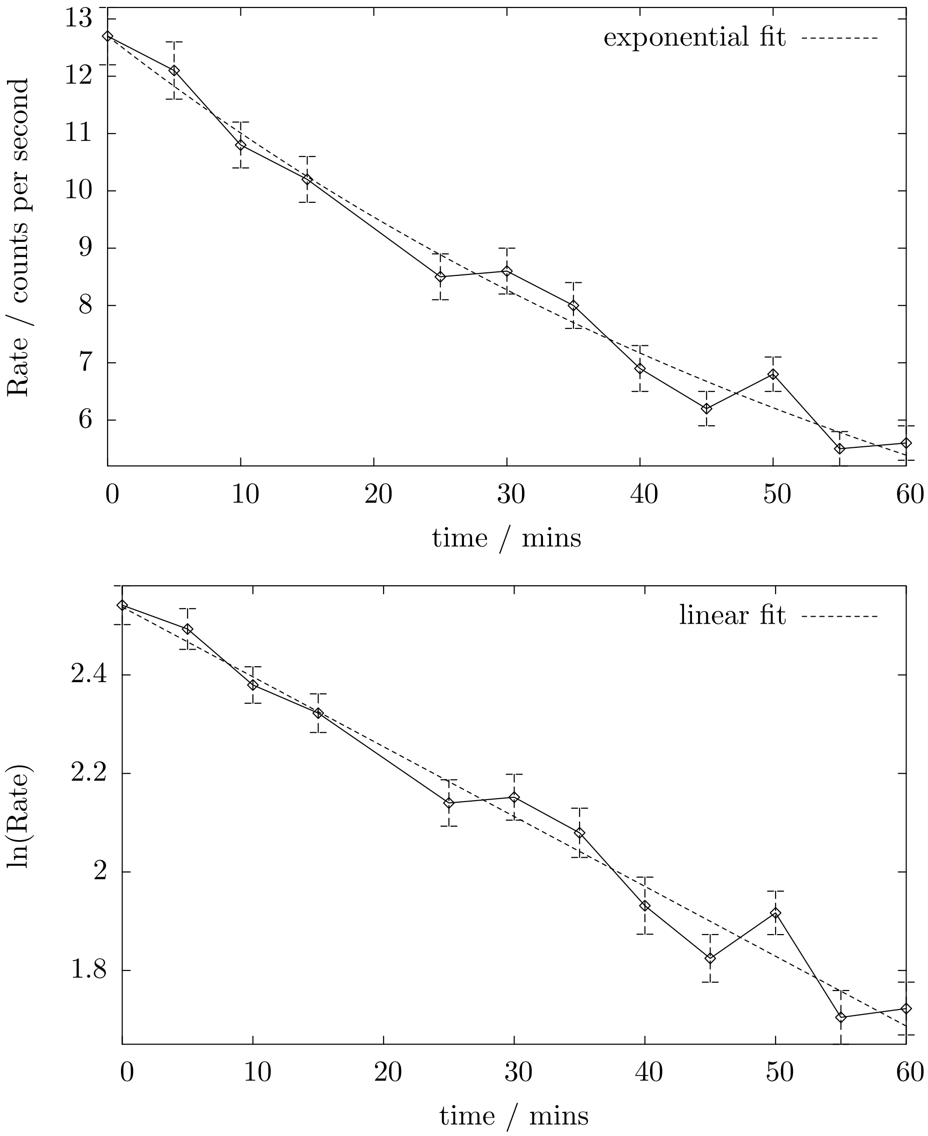

Figure 3: Rate vs. time is an exponential decay while the graph of ln(Rate) vs. time is linear.

In the case of the exponential fit $\lambda = .0143 \pm .0006$ and in the linear fit $\lambda = .0142 \pm .0007$. From the expression for half-life we have $t_{1/2} = 48 \pm 2$ minutes and $t_{1/2} = 49 \pm 2$ minutes for the half-life of radiactive indium.

3.2 Dead time of Geiger tube

Solving the formula for dead time we have

$$\begin{aligned}
\tau &= \frac{1}{m} \pm \sqrt{\frac{1}{m^2} - \frac{m_1 + m_2 - m}{m_1 m_2 m}} \\
\tau &= \frac{1}{785} \pm \sqrt{\frac{1}{(785)^2} - \frac{543 + 403 - 785}{785 \times 543 \times 403}} \\
\tau &= 2.1 \times 10^{-3}\text{s} \quad \text{or} \quad 4.5 \times 10^{-4}\text{s}
\end{aligned}$$

If these answers are checked by working out the true counts and checking they are both positive we find that $\tau = 4.5 \times 10^{-4}$ is the correct answer. This is of the order of dead time quoted in [1].

Working out the error in τ is quite tedious. Letting

$$A = \frac{1}{m^2} - \frac{m_1 + m_2 - m}{m m_1 m_2}$$

we have

$$\Delta A = \frac{2\Delta m}{m^3} + \left(\frac{\Delta m}{m} + \frac{\Delta m_1}{m_1} + \frac{\Delta m_2}{m_2} + \frac{\Delta m_1 + \Delta m_2 + \Delta m}{m_1 + m_2 - m}\right)\left(\frac{m_1 + m_2 - m}{m_1 m_2 m}\right)$$

so for τ

$$\begin{aligned}
\Delta\tau &= \frac{\Delta m}{m^2} + \frac{\Delta A}{2\sqrt{A}} \\
\Delta\tau &= .2 \times 10^{-4}\text{s}
\end{aligned}$$

4 Conclusions

The half-life of the radiactive indium was found to be 49 ± 2 minutes taking both methods of analysis into account.

The dead time of the Geiger tube was measured to be $\tau = (4.5 \pm .2) \times 10^{-4}$s.

References

[1] C. F. G. Delaney and E. C. Finch. *Radiation Detectors*. Oxford University Press.