MAU34601 Practical Numerical Simulations
Assignment 4 due 11/01/2023

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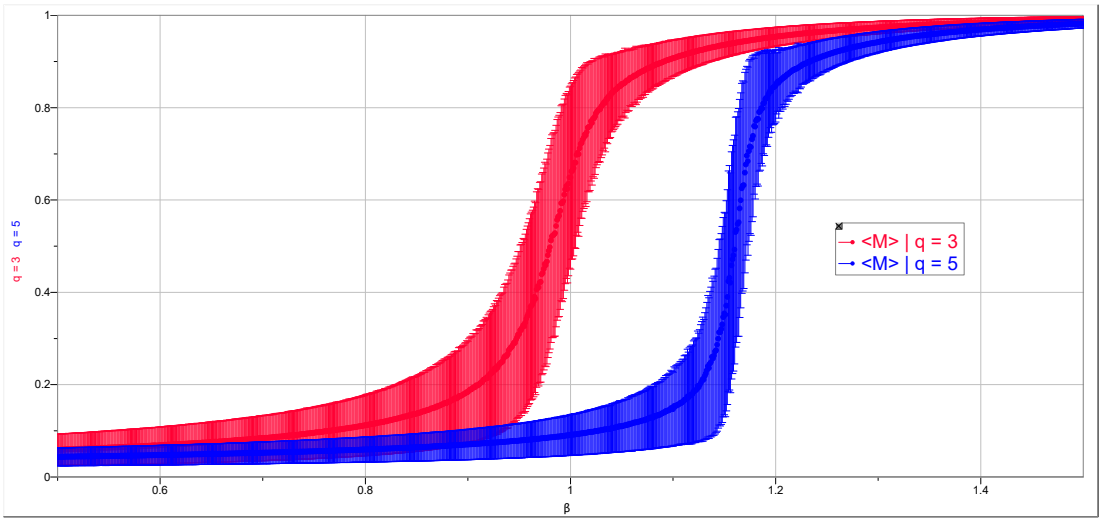
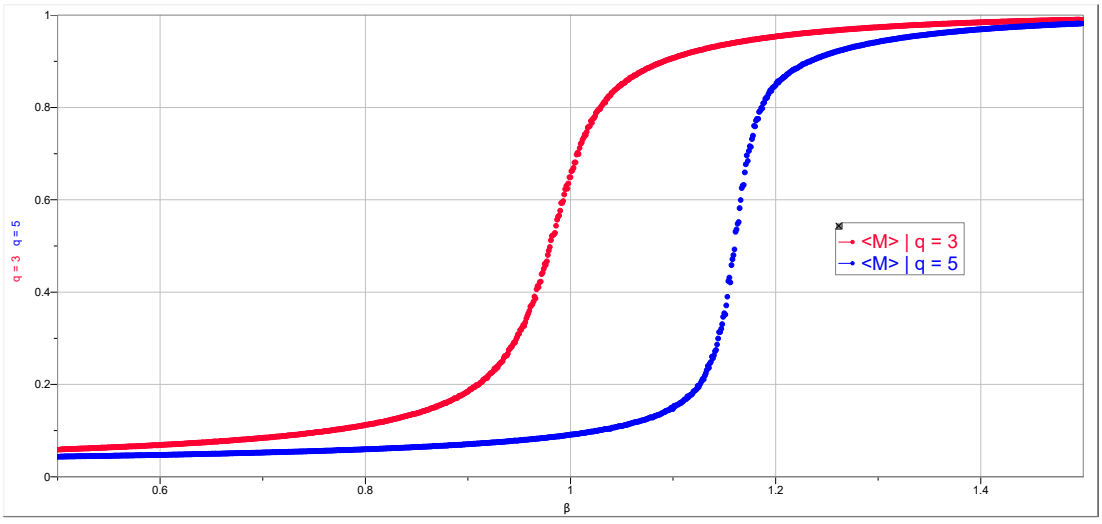
SS Theoretical Physics

1 The Potts model in two dimensions

The mean and standard deviation of the fractional magnetisation were calculated for the Potts model using the Metropolis algorithm for $q = 3, 5$, $\beta \in [0.5, 1.5]$ (Figure 1).



(a) Errorbars indicate one standard deviation.



(b) No errorbars.

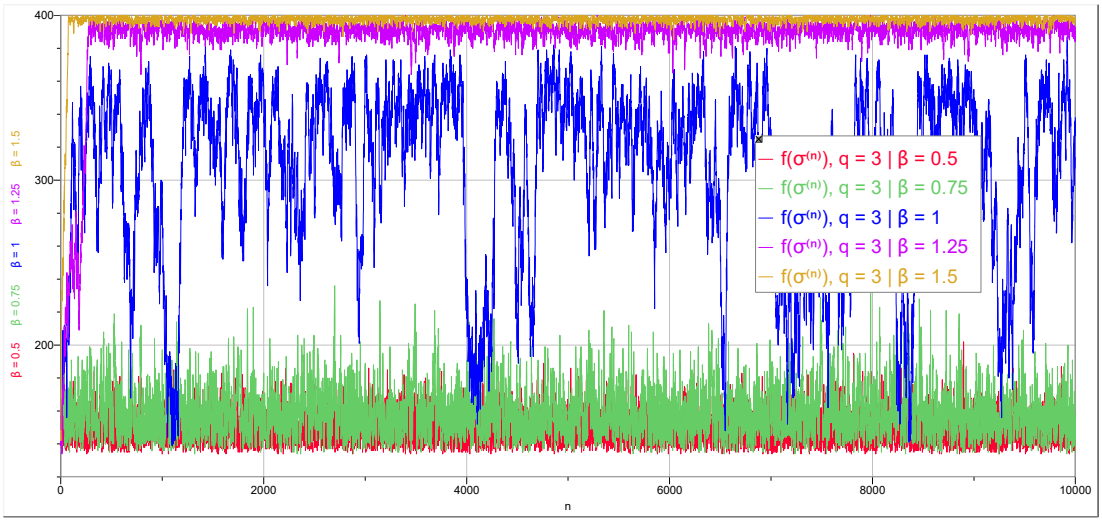
Figure 1: Plots of $\langle M \rangle$ for $q = 3, 5$, $\beta \in [0.5, 1.5]$.

From Figure 1 we can see that

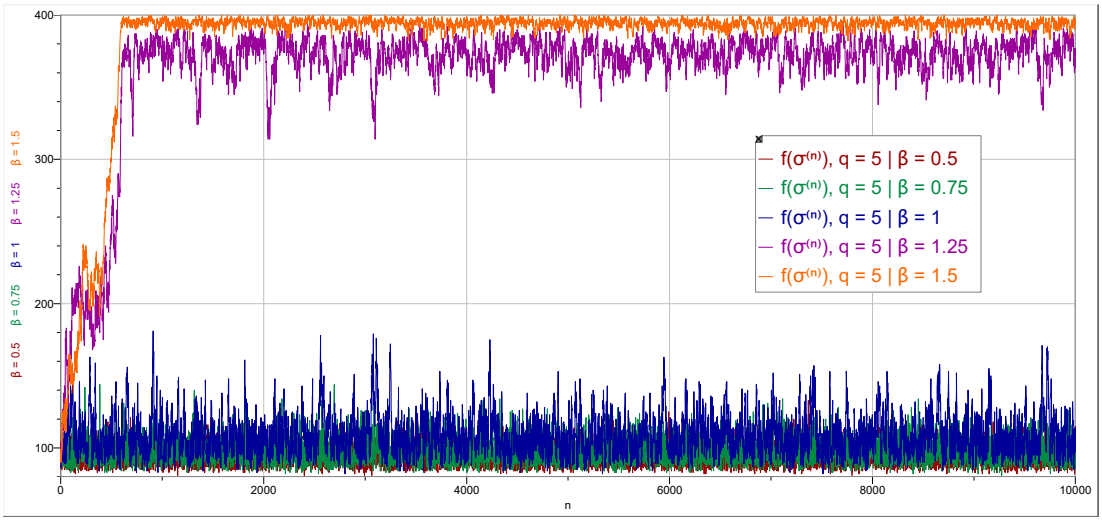
$$\lim_{\beta \to 0} M = 0, \qquad \lim_{\beta \to \infty} M = 1$$

for both $q = 3, 5$, i.e. high temperatures correspond to disordered states and low temperatures to ordered states. We can also see that the standard deviation is largest for both $q = 3, 5$ near $M = 0.5$, highlighting the phase transition at this point.

For each q and β, 510,000 iterations of the Metropolis algorithm were carried out. To deal with autocorrelation of the algorithm, the initial 10,000 iterations were discarded from calculations, and $f(\sigma)$ was calculated at every 10 steps, for a total of 50,000 data points. Below (Figure 2) are plots of $f(\sigma^n)$ for various values of q and β illustrating that the algorithm is sampling from the equilibrium distribution after 10,000 discarded iterations.



(a) $q = 3$.



(b) $q = 5$.

Figure 2: Plots of $f(\sigma^n)$ for various q, β.