

MAU22101: Group Theory

Assignment 1 due 19/10/2020

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I have read and I understand the plagiarism provisions in the General Regulations of the University Calendar for the current year, found at <http://www.tcd.ie/calendar>.

I have completed the Online Tutorial in avoiding plagiarism 'Ready, Steady, Write', located at <http://tcd-ie.libguides.com/plagiarism/ready-steady-write>.

Exercise 1

$$\begin{aligned}\sigma &= (123456) \\ &= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 5 & 6 & 1 \end{pmatrix}\end{aligned}$$

$$\begin{aligned}\sigma^2 &= \sigma\sigma \\ &= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 5 & 6 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 5 & 6 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 1 & 2 \end{pmatrix} \\ \sigma^2 &= (135)(246)\end{aligned}$$

$$\begin{aligned}\sigma^3 &= \sigma\sigma^2 \\ &= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 5 & 6 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 1 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 1 & 2 & 3 \end{pmatrix} \\ \sigma^3 &= (14)(25)(36)\end{aligned}$$

Exercise 2

$$\begin{aligned}\tau^{-1}(\tau(i_q)) &= i_q, \quad 1 \leq q \leq n \\ \implies \sigma\tau^{-1}(\tau(i_q)) &= \sigma(i_q) \\ &= \begin{cases} i_{q+1} & 1 \leq q < j \\ i_1 & q = j \\ i_q & j < q \leq n \end{cases} \\ \implies \tau\sigma\tau^{-1}(\tau(i_q)) &= \begin{cases} \tau(i_{q+1}) & 1 \leq q < j \\ \tau(i_1) & q = j \\ \tau(i_q) & j < q \leq n \end{cases}\end{aligned}$$

$\tau\sigma\tau^{-1}$ cycles through $\tau(i_1), \tau(i_2), \dots, \tau(i_j)$ and fixes $\tau(i_{j+1}), \tau(i_{j+2}), \dots, \tau(i_n)$

$$\begin{aligned}\implies \tau\sigma\tau^{-1} &= (\tau(i_1) \dots \tau(i_j))(\tau(i_{j+1})) \dots (\tau(i_n)) \\ &= (\tau(i_1) \dots \tau(i_j))\end{aligned}$$

Exercise 3

Cycle Decompositions	Permutations
$(1)(2)(3)(4)$	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$
$(1)(2)(34)$	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 4 & 3 \end{pmatrix}$
$(1)(3)(24)$	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 3 & 2 \end{pmatrix}$
$(1)(4)(23)$	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix}$
$(2)(3)(14)$	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 3 & 1 \end{pmatrix}$
$(2)(4)(13)$	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$
$(3)(4)(12)$	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 3 & 4 \end{pmatrix}$
$(1)(234)$	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}$
$(1)(243)$	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 2 & 3 \end{pmatrix}$
$(2)(134)$	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 4 & 1 \end{pmatrix}$
$(2)(143)$	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix}$
$(3)(124)$	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$
$(3)(142)$	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 3 & 2 \end{pmatrix}$
$(4)(123)$	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}$
$(4)(132)$	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix}$
$(12)(34)$	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$
$(13)(24)$	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$
$(14)(23)$	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$
(1234)	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$
(1243)	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$
(1324)	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$
(1342)	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$
(1423)	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$
(1432)	$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$