MAU11204: Analysis on the Real Line
Homework 10 due 21/03/2021

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I have read and I understand the plagiarism provisions in the General Regulations of the University Calendar for the current year, found at http://www.tcd.ie/calendar.
I have completed the Online Tutorial in avoiding plagiarism 'Ready, Steady, Write', located at http://tcd-ie.libguides.com/plagiarism/ready-steady-write.

Problem 1

We can rewrite $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{1}{k+n}$ as $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{1}{\frac{k}{n}+1}\frac{1}{n}$. This takes the form of a limit of a Riemann sum $\lim\limits_{n\to\infty}R(f,P_n,S_n)=\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}f(S_{n_k})\,(x_{n_k}-x_{n_{k-1}})$, where $f(x)=\frac{1}{x+1}$, $S_n=\left\{\frac{1}{n},\frac{2}{n},\ldots,\frac{n-1}{n},1\right\}$, and $P_n=\left\{0,\frac{1}{n},\frac{2}{n},\ldots,\frac{n-1}{n},1\right\}$. Since $S_{n_k}=\frac{k}{n}\in\left[\frac{k-1}{n},\frac{k}{n}\right]=[x_{n_{k-1}},x_{n_k}]$, S is an adequate choice of sample points for the partition P of $[0,1]$. Since the set of discontinuities of f on $[0,1]$ is negligible (as f is continuous on $[0,1]$), then f is integrable. Since $\lim\limits_{n\to\infty}||P_n||=\lim\limits_{n\to\infty}\frac{1}{n}=0$, and $f:[0,1]\to\mathbb{R}$ is integrable, we have that

$$\begin{aligned}
\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{k+n} &= \lim_{n\to\infty}R(f,P_n,S_n)\\
&= \int_a^b f(x)\,dx\\
&= \int_0^1 \frac{1}{x+1}\,dx\\
&= \ln(2)-\ln(0)
\end{aligned}$$

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{k+n} = \ln(2)$$