MAU23401: Advanced Classical Mechanics II
Homework 9 due 09/04/2021

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Problem 1

(i)

$$\begin{aligned}
K &= \sum_{j=1}^{N} \frac{m_j}{2}\dot{y}_j^2 \\
U &= \sum_{j=1}^{N+1} T_j l_j \\
&= \sum_{j=0}^{N} T_j \sqrt{(y_{j+1}-y_j)^2 + a^2} \\
L &= K - U \\
L &= \sum_{j=0}^{N} \left(\frac{m_j}{2}\dot{y}_j^2 - T_j \sqrt{(y_{j+1}-y_j)^2 + a^2} \right)
\end{aligned}$$

(ii)

$$\begin{aligned}
l_j &= \sqrt{(y_{j+1}-y_j)^2 + a^2} \\
&= a\sqrt{1 + \left(\frac{y_{j+1}-y_j}{a}\right)^2} \\
&\approx a\left(1 + \frac{1}{2}\left(\frac{y_{j+1}-y_j}{a}\right)^2\right) \\
\implies L &= \sum_{j=0}^{N} \left(\frac{m_j}{2}\dot{y}_j^2 - aT_j - \frac{aT_j}{2}\left(\frac{y_{j+1}-y_j}{a}\right)^2 \right) \\
L &= \frac{1}{2}\sum_{j=0}^{N} a\left(\frac{m_j}{a}\dot{y}_j^2 - T_j\left(\frac{y_{j+1}-y_j}{a}\right)^2 \right)
\end{aligned}$$

(iii)

$a \to 0 \implies \dfrac{y_{j+1}-y_j}{a} \to y_j'$, $T_j \to T$, $\dfrac{m_j}{a} \to \mu$

$$\begin{aligned}
\lim_{a\to 0} L &= \frac{1}{2}\lim_{a\to 0}\sum_{j=0}^{N} a\left(\mu\dot{y}_j^2 - T y_j'^2\right) \\
\lim_{a\to 0} L &= \frac{1}{2}\int_0^N dx\left(\mu\dot{y}^2 - T y'^2\right), \quad x_1, x_2 = 0, N
\end{aligned}$$

(iv)

$$\frac{\partial \mathcal{L}}{\partial \dot{y}} = \mu \dot{y} \qquad \frac{\partial \mathcal{L}}{\partial y'} = -Ty' \qquad \frac{\partial \mathcal{L}}{\partial y} = 0$$

$$\implies \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{y}} = \mu' \dot{x}\dot{y} + \mu \ddot{y} \qquad \implies \frac{d}{dx}\frac{\partial \mathcal{L}}{\partial y'} = -T'y' - Ty''$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{y}} + \frac{d}{dx}\frac{\partial \mathcal{L}}{\partial y'} + \frac{\partial \mathcal{L}}{\partial y} = 0$$

$$\mu' \dot{x}\dot{y} + \mu \ddot{y} - T'y' - Ty'' = 0$$