

MAU23401: Advanced Classical Mechanics II

Homework 9 due 09/04/2021

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I have read and I understand the plagiarism provisions in the General Regulations of the University Calendar for the current year, found at <http://www.tcd.ie/calendar>.

I have completed the Online Tutorial in avoiding plagiarism 'Ready, Steady, Write', located at <http://tcd-ie.libguides.com/plagiarism/ready-steady-write>.

Problem 1

(i)

$$\begin{aligned} K &= \sum_{j=1}^N \frac{m_j}{2} \dot{y}_j^2 \\ U &= \sum_{j=1}^{N+1} T_j l_j \\ &= \sum_{j=0}^N T_j \sqrt{(y_{j+1} - y_j)^2 + a^2} \\ L &= K - U \\ L &= \sum_{j=0}^N \left(\frac{m_j}{2} \dot{y}_j^2 - T_j \sqrt{(y_{j+1} - y_j)^2 + a^2} \right) \end{aligned}$$

(ii)

$$\begin{aligned} l_j &= \sqrt{(y_{j+1} - y_j)^2 + a^2} \\ &= a \sqrt{1 + \left(\frac{y_{j+1} - y_j}{a} \right)^2} \\ &\approx a \left(1 + \frac{1}{2} \left(\frac{y_{j+1} - y_j}{a} \right)^2 \right) \\ \Rightarrow L &= \sum_{j=0}^N \left(\frac{m_j}{2} \dot{y}_j^2 - a T_j - \frac{a T_j}{2} \left(\frac{y_{j+1} - y_j}{a} \right)^2 \right) \\ L &= \frac{1}{2} \sum_{j=0}^N a \left(\frac{m_j}{a} \dot{y}_j^2 - T_j \left(\frac{y_{j+1} - y_j}{a} \right)^2 \right) \end{aligned}$$

(iii)

$$a \rightarrow 0 \Rightarrow \frac{y_{j+1} - y_j}{a} \rightarrow y'_j, T_j \rightarrow T, \frac{m_j}{a} \rightarrow \mu$$

$$\begin{aligned} \lim_{a \rightarrow 0} L &= \frac{1}{2} \lim_{a \rightarrow 0} \sum_{j=0}^N a (\mu \dot{y}_j^2 - T y_j'^2) \\ \lim_{a \rightarrow 0} L &= \frac{1}{2} \int_0^N dx (\mu \dot{y}^2 - T y'^2), \quad x_1, x_2 = 0, N \end{aligned}$$

(iv)

$$\begin{array}{lll} \frac{\partial \mathcal{L}}{\partial \dot{y}} = \mu \dot{y} & \frac{\partial \mathcal{L}}{\partial y'} = -T y' & \frac{\partial \mathcal{L}}{\partial y} = 0 \\ \implies \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{y}} = \mu' \dot{x} \dot{y} + \mu \ddot{y} & \implies \frac{d}{dx} \frac{\partial \mathcal{L}}{\partial y'} = -T' y' - T y'' & \end{array}$$

$$\begin{array}{l} \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{y}} + \frac{d}{dx} \frac{\partial \mathcal{L}}{\partial y'} + \frac{\partial \mathcal{L}}{\partial y} = 0 \\ \mu' \dot{x} \dot{y} + \mu \ddot{y} - T' y' - T y'' = 0 \end{array}$$