Pion-Pion Scattering in the Non-Linear Sigma Model

Final Year Project Proposal

Ruaidhrí Campion
Supervised by Prof. Mike Peardon

11 October 2022

Background

This project will investigate the $SU(2)$ non-linear sigma model [1] using lattice quantum field theory techniques. This model is an effective description of the physics of pions, including their scattering in collider experiments, at low energies.

The project will study pion-pion scattering by computing the energy spectrum of the sigma model using numerical simulation techniques. Quantum field theories on the lattice can be investigated on the computer using Monte Carlo integration to evaluate the lattice representation of the path integral.

Objectives

For this work, the fields in the path integral will be elements of the $SU(2)$ Lie group at each site of the lattice. A theory with an $SU(2)_L \times SU(2)_R$ chiral symmetry can be defined in which the pions interact and scatter. Markov Chain Monte Carlo (MCMC) code will be written to evaluate expectation values in the theory and to find the energy spectrum of the theory in a finite volume. To find this spectrum, the resulting statistical data will be analysed and fits evaluated to extract energies. The autocorrelations found in Markov chain methods will be investigated to check the validity of the analysis and its statistical uncertainties.

The energy spectrum of the system in finite volume gives information on scattering properties of the theory via the Luescher method [2]. The final goal of the project will be to investigate pion-pion scattering in the theory and map the phase shift for scattering for a range of pion momenta.

If time permits, the model will be extended to include the coupling of the pions to a heavy scalar boson and the decay of this boson into pions will be studied.

Tasks

- To start off the project, I will have to refresh my memory on the theory and implementation of MCMC methods and statistical analysis by revising the references I used for my summer project [3, 4], as well as those recommended by my supervisor [5, 6].
- Next, I will simulate a scalar field over a d-dimensional lattice using the Metropolis-Hastings algorithm and compare the analytic and calculated correlations between sites, to make sure I understand the basics of my project before I start simulating more complicated (and less solvable) systems.
- Once I have done this, I will need to learn how to sample elements from $SU(2)$ using Monte Carlo methods [6] and read some literature on lattice gauge theories [7].
- With all of these necessary prerequisites, I will finally be able to simulate the sigma model, the field theory at the heart of the project. With this I will be able to carry out the aforementioned objectives, i.e. calculate energy spectra, scattering phase shifts, and statistical uncertainties.

References

[1] M. Gellmann and M. Lévy, "The Axial Vector Current in Beta-Decay," *Nuovo Cimento*, vol. 16, no. 4, pp. 705–726, 1960.

[2] R. A. Briceno, J. J. Dudek, and R. D. Young, "Scattering Processes and Resonances from Lattice QCD," *Reviews of Modern Physics*, vol. 90, no. 2, p. 42, 2018.

[3] I. Sachs, S. Sen, and J. C. Sexton, *Elements of Statistical Mechanics: With an Introduction to Quantum Field Theory and Numerical Simulation.* Cambridge: Cambridge University Press, 2006.

[4] A. D. Sokal, "Monte Carlo Methods in Statistical Mechanics: Foundations and Algorithms," 1996.

[5] S. M. Ross, *Simulation.* Cambridge: Academic Press, 5th ed., 2005.

[6] D. Landau and K. Binder, *A Guide to Monte Carlo Simulations in Statistical Physics.* Cambridge: Cambridge University Press, 5th ed., 2021.

[7] H. J. Rothe, *Lattice Gauge Theories: An Introduction.* World Scientific Lecture Notes in Physics, Singapore: World Scientific Publishing Company, 3rd ed., 2005.