Experimental Lab Report - RC Filter Networks

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Contents

1 Abstract

In this experiment, RC filter networks were examined. A high-pass filter was tested to see if only high frequencies were allowed through, and a low-pass filter was tested to see if only low frequencies were allowed through, while blocking other frequencies.

The filters were found to work as expected. The half-power points of the high-pass filter and low-pass filter were found to be 1202 ± 28 Hz and 1000 ± 23 Hz, respectively. The phase differences at the half-power point for the high-pass filter and low-pass filter were found to be 0.67 ± 0.01 rad and 0.89 ± 0.01 rad, respectively.

2 Introduction

An RC filter circuit is a circuit consisting of resistors and capacitors. As the name suggests, RC circuits filter certain frequencies, allowing signals of some frequencies to pass but blocking others.

Two common examples of RC circuits are the high-pass filter and the low-pass filter. A basic high-pass filter consists of a capacitor in series and a resistor in parallel, whereas a basic low-pass filter consists of a resistor in series and a capacitor in parallel.

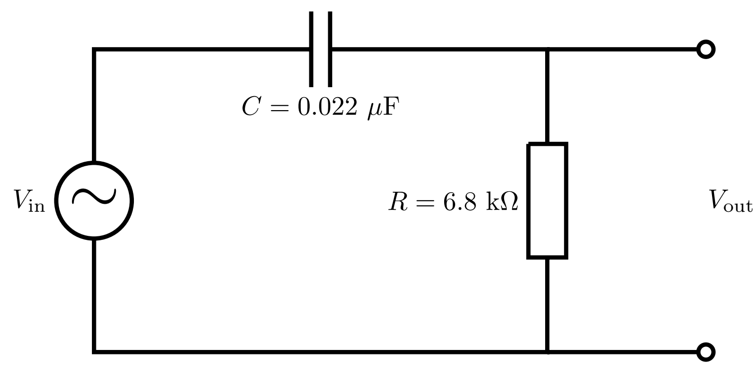

Figure 1: an example of a high-pass filter

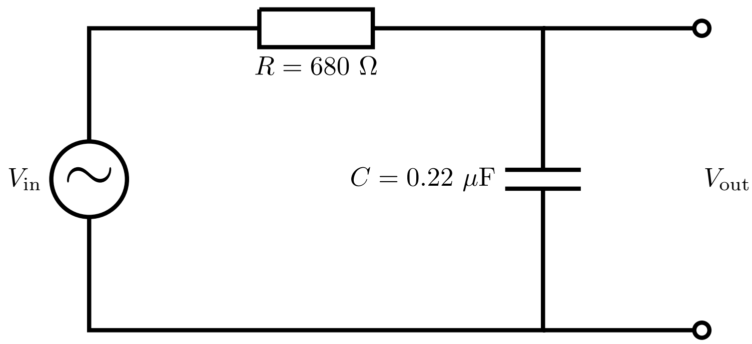

Figure 2: an example of a low-pass filter

The ratio of two signals, in decibels, is defined as[1]

$$10 \log_{10} \left(\frac{P_{\text{out}}}{P_{\text{in}}} \right),$$

where P_{in} and P_{out} represent the power of the input and output voltage, respectively. If the ratio of the two powers is $\frac{1}{2}$, then the ratio above is given by

$$10 \log_{10} \frac{1}{2} \approx -3.$$

It is for this reason that the half-power point (or cutoff frequency) f_c is also called the 3 dB point, as a drop in 3 dB corresponds to a halving of the power.

The time constant of the circuit τ is defined as[2]

$$\tau = RC.$$

The half-power point can be expressed in terms of the time constant as

$$\begin{aligned} f_c &= \frac{1}{2\pi\tau} \\ &= \frac{1}{2\pi RC}. \end{aligned}$$

The phase difference between the input and output waves ϕ is given by

$$\phi = \arctan \left(\frac{1}{2\pi f RC} \right).$$

for a high-pass filter[3] and

$$\phi = \arctan \left(2\pi f RC \right).$$

for a low-pass filter.[4] When $f = f_c$, then the phase difference is simply $\arctan 1$, which is equal to $\frac{\pi}{4}$ rad or ~ 0.785 rad.

[1] Horowitz, Hill; 2015
[2] Young, Freedman; 2016
[3] ElectronicsTutorials, 2020
[4] ElectronicsTutorials, 2020

3 Experimental Method

1. Set up the apparatus as shown in Figure 1, with the channels of the oscilloscope measuring V_{in} over the oscillator and V_{out} over the entire circuit. Ensure that the earths on the oscilloscope and oscillator are connected together.

2. Vary the frequency on the oscillator to find the region in which the peak-to-peak input voltage V_{in} remains constant.

3. Measure and record the phase difference between the two waves ϕ and the peak-to-peak output voltage V_{out} for about 20 different frequencies across the range.

4. Plot a graph of $\log_{10}\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right)$ against $\log_{10} f$ and also a graph of ϕ against $\log_{10} f$.

5. Determine the half-power point of the filter, and the phase difference at this frequency.

6. Set up the apparatus as shown in Figure 2, again ensuring that the earths are connected together.

7. Find out how V_{out} depends on f.

8. Measure the half-power point and the phase difference at this frequency.

9. Apply a square wave from the oscillator to Figure 1.

10. By taking three values for the frequency such that $T \ll RC$, $T \approx 2RC$ and $T \gg RC$, where T is the period of the square wave, measure the time constant τ using the formula $V_{\text{out}} = V_{\text{in}}\left(1 - e^{-\frac{t}{\tau}}\right)$.

11. Repeat steps 9-10 for Figure 2.

4 Results

4.1 High-Pass Filter

The peak-to-peak input voltage V_{in} was kept constant at 21.5 ± 0.2 V.

The following data was obtained by varying the frequency f:

Frequency f, in Hz	Peak-to-Peak Output Voltage V_{out}, in V	Time Delay t, in μs	Phase Difference ϕ, in rad
58.75 ± 0.01	1.20 ± 0.1	4000 ± 200	1.48 ± 0.08
72.21 ± 0.01	1.46 ± 0.1	3200 ± 200	1.45 ± 0.10
97.01 ± 0.01	2.00 ± 0.2	2400 ± 200	1.46 ± 0.13
120.5 ± 0.1	2.48 ± 0.2	1800 ± 100	1.36 ± 0.08
143.9 ± 0.1	2.94 ± 0.2	1520 ± 40	1.37 ± 0.04
169.2 ± 0.1	3.42 ± 0.2	1280 ± 40	1.36 ± 0.05
180.8 ± 0.1	3.64 ± 0.2	1200 ± 40	1.36 ± 0.05
200.5 ± 0.1	4.02 ± 0.2	1080 ± 40	1.36 ± 0.05
228.4 ± 0.1	4.60 ± 0.4	920 ± 40	1.32 ± 0.06
272.2 ± 0.1	5.36 ± 0.4	760 ± 40	1.30 ± 0.07
298.7 ± 0.1	5.84 ± 0.4	680 ± 20	1.28 ± 0.04
309.2 ± 0.1	6.04 ± 0.4	640 ± 20	1.24 ± 0.04
412.9 ± 0.1	7.76 ± 0.4	460 ± 20	1.19 ± 0.06
511.1 ± 0.1	9.28 ± 0.8	340 ± 20	1.09 ± 0.07
668.4 ± 0.1	11.1 ± 0.1	230 ± 10	0.97 ± 0.05
871.9 ± 0.1	13.1 ± 0.1	160 ± 10	0.88 ± 0.06
996.8 ± 0.1	14.0 ± 0.1	130 ± 10	0.81 ± 0.07
1193 ± 1	15.2 ± 0.1	90 ± 10	0.68 ± 0.08
1422 ± 1	16.2 ± 0.1	70 ± 10	0.63 ± 0.09
1736 ± 1	17.2 ± 0.2	48 ± 4	0.52 ± 0.05
2106 ± 1	18.4 ± 0.2	36 ± 4	0.48 ± 0.06
2414 ± 1	18.8 ± 0.2	26 ± 2	0.40 ± 0.03
2805 ± 1	19.2 ± 0.2	20 ± 2	0.35 ± 0.04
3383 ± 1	19.6 ± 0.2	16 ± 2	0.34 ± 0.05
3503 ± 1	19.6 ± 0.2	14 ± 2	0.31 ± 0.05
4003 ± 1	19.8 ± 0.2	10 ± 2	0.25 ± 0.05
5200 ± 1	20 ± 0.2	5 ± 1	$0.16 \pm 0.04^{*}$

The following graphs of $\log_{10}\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right)$ against $\log_{10} f$ and ϕ against $\log_{10} f$ were plotted from the data:

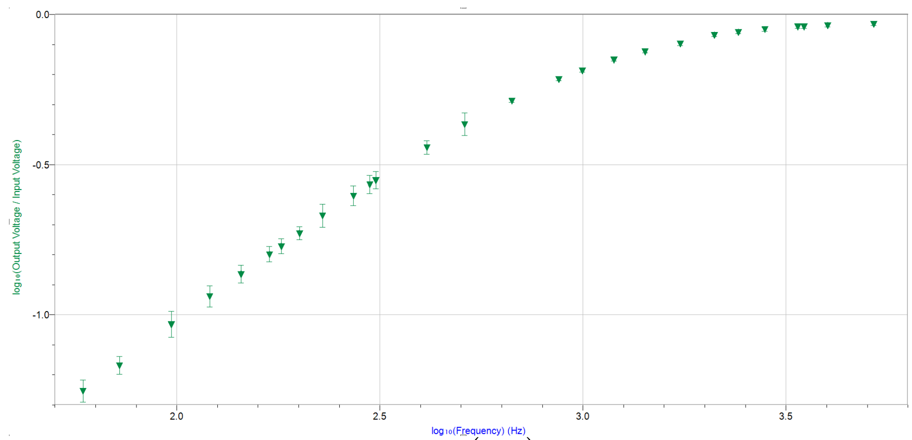

Figure 3: plot of $\log_{10}\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right)$ against $\log_{10} f$

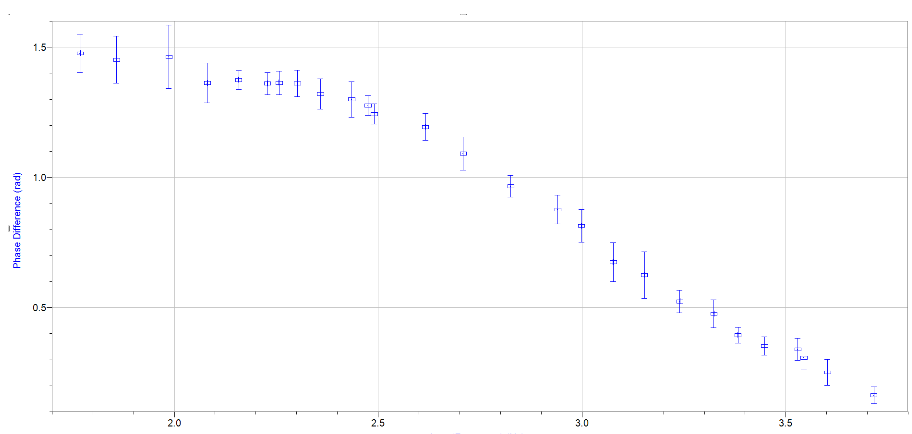

Figure 4: plot of ϕ against $\log_{10} f$

Using the interpolate feature, the value of of $\log_{10} f_c$ at $\log_{10}\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right) = \log_{10} \frac{1}{\sqrt{2}} \approx -0.1505$ was measured to be 3.08 ± 0.01 Hz, and thus the value of f_c was calculated to be 1202 ± 28 Hz. The phase difference at the half-power point ϕ_c was measured to be 0.67 ± 0.01 rad.

4.2 Low-Pass Filter

The peak-to-peak input voltage V_{in} was kept constant at 21.6 ± 0.2 V.

The following data was obtained by varying the frequency f:

Frequency f, in Hz	Peak-to-Peak Output Voltage V_{out}, in V	Time Delay t, in μs	Phase Difference ϕ, in rad
6.91 ± 0.01	21.6 ± 0.2	8 ± 4	0.0003 ± 0.0002
71.12 ± 0.01	21.4 ± 0.2	800 ± 40	0.3575 ± 0.0179
398.8 ± 0.1	21.2 ± 0.2	300 ± 20	0.7517 ± 0.0501
812.4 ± 0.1	18.4 ± 0.2	170 ± 10	0.8678 ± 0.0510
1009 ± 1	15.2 ± 0.2	140 ± 10	0.8876 ± 0.0634
2509 ± 1	6.8 ± 0.2	70 ± 10	1.1035 ± 0.1576

The following graphs of $\log_{10}\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right)$ against $\log_{10} f$ and ϕ against $\log_{10} f$ were plotted from the data:

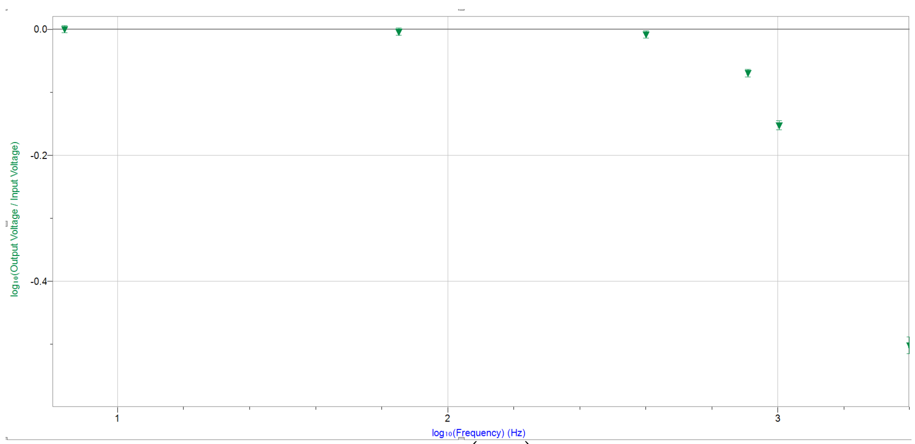

Figure 5: plot of $\log_{10}\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right)$ against $\log_{10} f$

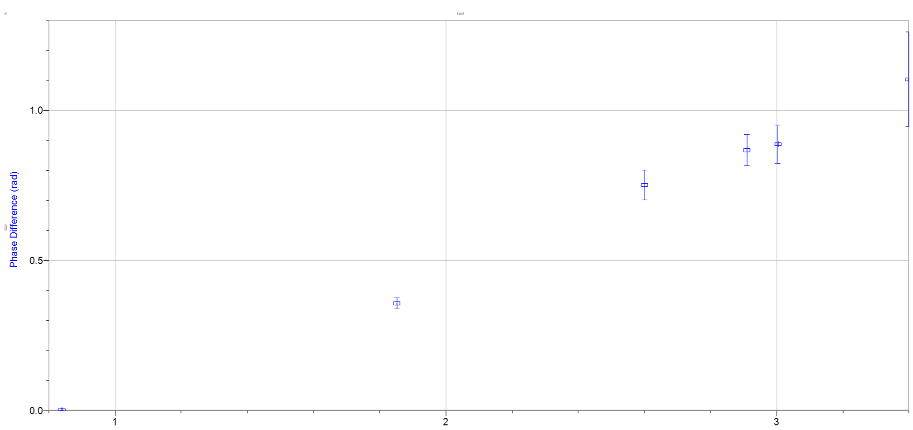

Figure 6: plot of ϕ against $\log_{10} f$

Using the interpolate feature, the value of of $\log_{10} f_c$ at $\log_{10}\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right) = \log_{10}\frac{1}{\sqrt{2}} \approx -0.1505$ was measured to be 3.00 ± 0.01 Hz, and thus the value of f_c was calculated to be 1000 ± 23 Hz. The phase difference at the half-power point ϕ_c was measured to be 0.89 ± 0.01 rad.

4.3 Square Wave Response

Results were not obtained for this section - see discussion for explanation.

5 Discussion

Using the high-pass filter, the ratio of the output voltage to the input voltage was close to 0 for small frequencies and close to 1 for large frequencies, and the phase difference tended to 0 as the frequency increased. Using the low-pass filter, the ratio of the output voltage to the input voltage was close to 1 for small frequencies and close to 0 for large frequencies, and the phase difference increased from 0 as frequency increased from 0. These results are as expected, as they show that high-pass filters tend to block low frequency signals, and low-pass filters tend to block high-frequency signals.

For the high-pass filter, the half-power point was found to be 1202 ± 28 Hz, and the phase difference was found to be 0.67 ± 0.01 rad. For the low-pass filter, the half-power point was found to be 1000 ± 23 Hz, and the phase difference was found to be 0.89 ± 0.01 rad. These results, within experimental error, agree with the theoretical values of the half-power point of $\frac{1}{2\pi(6.8\times10^3)(0.022\times10^{-6})} = \frac{1}{2\pi(680)(0.22\times10^{-6})} \approx 1064$ Hz and the corresponding phase difference of $\frac{\pi}{4} \approx 0.785$ rad.

One reason why logarithmic axes are preferable in this experiment is because of the densely packed values for the frequency and ratios of voltages. Plotting a graph of $\dfrac{V_{\text{out}}}{V_{\text{in}}}$ against f or ϕ against f would result in a high density of points near $f = 0$ because of the high values of frequencies recorded, particularly for the high-pass filter. Using a log-log plot helps to space the data out.

Another reason why logarithmic axes were used in this experiment was so that the relationships between the data could be recognised easier. For example, for the high-pass filter, it was quite simple to notice that $\log_{10}\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right)$ tended to 0 as $\log_{10} f$ increased. It was thus deduced that the ratio of the voltages tends to 1 as frequency increases. This relationship would be harder to spot on a plot of $\dfrac{V_{\text{out}}}{V_{\text{in}}}$ against f.

Results were not obtained for section 3 of this experiment. This was mainly due to time constraints. The original data obtained for sections 1 and 2 was over a very short range of the frequencies, and so the plot of $\log_{10}\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right)$ against $\log_{10} f$ simply showed a linear relationship. For this reason, sections 1 and 2 had to be repeated, which resulted in a short amount of time remaining for section 3. Section 3 was originally attempted but was not finished due to an unknown error and a short amount of time to find and fix this error. The most likely cause of the error was the possibility that the circuits were set up incorrectly, due to having to change them twice in a short amount of time.

If the correct data was obtained for this section, the time at which $V_{\text{out}} = V_{\text{in}}\left(1 - e^{-1}\right) \approx 0.63 V_{\text{in}}$ would occur would be at $t = \tau = 1.496 \times 10^{-4}$ s $=$ 149.6 μs. This value for t is the same as the product RC, and so the expression $\tau = RC$ would have been verified.

The reason a high-pass filter is known as a differentiating circuit is because the output voltage is proportional to the derivative of the input voltage with respect to time:

$$
\begin{aligned}
I_C &= \frac{dQ_C}{dt} && \text{current definition} \\
&= \frac{d}{dt}(CV_C) && Q = VC \\
&= C\frac{dV_C}{dt} && \frac{dC}{dt} = 0 \\
&= C\frac{dV_{\text{in}}}{dt} && V_C = V_{\text{in}} \\
V_{\text{out}} &= V_R && \\
&= I_R R && V = IR \\
&= I_C R && I_R = I_C \\
V_{\text{out}} &= RC\frac{dV_{\text{in}}}{dt} &&
\end{aligned}
$$

$$
\begin{aligned}
\text{where } I_C,\ I_R &\equiv \text{current over the capacitor or resistor} \\
V_C,\ V_R &\equiv \text{voltage across the capacitor or resistor}
\end{aligned}
$$

The reason a low-pass filter is known as an integrating circuit is because the output voltage is proportional to the integral of the input voltage with respect to time:

$$\begin{aligned}
I_C &= C\frac{dV_C}{dt} && \text{from before} \\
&= C\frac{dV_{\text{out}}}{dt} && V_C = V_{\text{out}} \\
\implies V_{\text{out}} &= \frac{1}{C}\int I_C\,dt \\
&= \frac{1}{C}\int I_R\,dt && I_C = I_R \\
&= \frac{1}{C}\int \frac{V_R}{R}\,dt && V = IR \\
&= \frac{1}{RC}\int V_{\text{in}}\,dt && V_R = V_{\text{in}}
\end{aligned}$$

6 Error Analysis

The uncertainty in the frequency f was based on the precision of the frequency shown on the oscillator.

The uncertainties in the input and output voltages V_{in} and V_{out} were found by noting the fluctuation of these values as they appeared on the oscilloscope.

The uncertainty in the time delay was found by varying the time measure by the smallest possible increment and noting the change.

All other uncertainties were calculated using the formula

$$\begin{aligned}
\Delta f &= \sqrt{\sum_{i=1}^{n}\left(\frac{\partial f}{\partial x_i}\Delta x_i\right)^2} \\
\text{where } f &= f(x_1, x_2, \ldots, x_n) \\
\Delta q &\equiv \text{uncertainty in } q
\end{aligned}$$

*For example, the uncertainty in the phase difference ϕ was calculated as follows:

$$f = 5200 \text{ Hz} \qquad t = 5 \times 10^{-6} \text{ s}$$
$$\Delta f = 1 \text{ Hz} \qquad \Delta t = 1 \times 10^{-6} \text{ s}$$
$$\frac{\partial \phi}{\partial f} = 2\pi t \qquad \frac{\partial \phi}{\partial t} = 2\pi f$$

$$\begin{aligned}
\Delta\phi &= \sqrt{\left(\frac{\partial \phi}{\partial f}\Delta f\right)^2 + \left(\frac{\partial \phi}{\partial t}\Delta t\right)^2} \\
&= 2\pi\sqrt{(t\Delta f)^2 + (f\Delta t)^2} \\
&= 2\pi\sqrt{(5 \times 10^{-6} \times 1)^2 + (5200 \times 1 \times 10^{-6})^2} \\
&= 0.03267 \\
&\approx 0.04 \text{ rad} \\
\implies \phi &= 0.16 \pm 0.04 \text{ rad}
\end{aligned}$$

7 Conclusion

High-pass and low-pass filters are effective at attenuating the respective unwanted signals, as demonstrated in this experiment. Within experimental error, the half-power point and phase difference were found to agree with the theoretical values of 1064 Hz and $\frac{\pi}{4}$ rad, respectively.

8 References

1: P. Horowitz, W. Hill, *The Art of Electronics (3rd Edition)*, Cambridge University Press, Cambridge, 2015, ch. 1.3, p. 15.

2: H. D. Young, R. A. Freedman, *University Physics (14th Edition)*, Pearson Education, London, 2016, ch. 26.4, p. 888.

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accessed November 2020.

All figures are of my own making, either using Logger Pro or TikZ.