Experimental Lab Report - Craters of the Moon

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Contents

1 Abstract

In this experiment, images of craters on the surface of the Moon were analysed using computer software. A linear relationship was found between the height and diameter of the crater, and the kinetic energy and mass of the impacting body that caused each crater was calculated. It was found that the majority of the Moon's craters are <16 km wide. The ratio of craters in the maria to craters in the highland regions was found to be approximately 2:7.

2 Introduction

Theories of how the Solar System was formed have varied greatly over time. For example, ancient Egyptians, Greeks and Romans believed that the Sun was created by the gods Ra,[1] Helios,[2] and Sol,[3] respectively. More modern theories have more scientific support, however. One theory, proposed by Georges Leclerc de Buffon in 1749, states that a passing object, such as a comet, struck the Sun, and the scattered material formed the Solar System.[4] Another theory, originally conceived by Rene Descartes in 1644, states that the Solar System was formed as a result of the Sun's formation.[4] Today, the second theory, also known as the nebular theory, is more commonly accepted, as it is has been concluded that comets are far too small to have formed the Solar System under Buffon's theory.[5] It is believed that a rotating cloud of gas and dust collapsed and formed the Sun, and the remnants of this cloud formed the Solar System.[4].

In 1665, Robert Hooke put forward two theories as to how lunar craters are formed; one suggested that projectile impact was the cause of the craters, and the other suggested that volcanic activity was the cause.[6] Today it is widely accepted that the craters are a result of impacting bodies left over from the Solar System's formation.

The Moon consists of different shaded regions. The dark regions are called maria (after the Latin word *mare* for sea), and the bright regions are called highland regions (due to the mountainous terrain of these regions). The maria are very new areas compared to the highland regions. After bodies collided with the Moon and formed craters, volcanic episodes caused magma to rise to the surface and fill some of these craters. The cooling and solidifying of the magma resulted in dark, flat sections of the Moon.[7] Today, there are far fewer craters in the maria than in the highland regions as there has been less time for craters to form here.

[1] Britannica, 2020
[2] Britannica, 2020
[3] Britannica, 2020
[4] Kutner, 2003
[5] Woolfson, 1993
[6] Hooke, 1665
[7] European Space Agency, 2020

This experiment involves analysing various images of the Moon's surface taken by satellites.

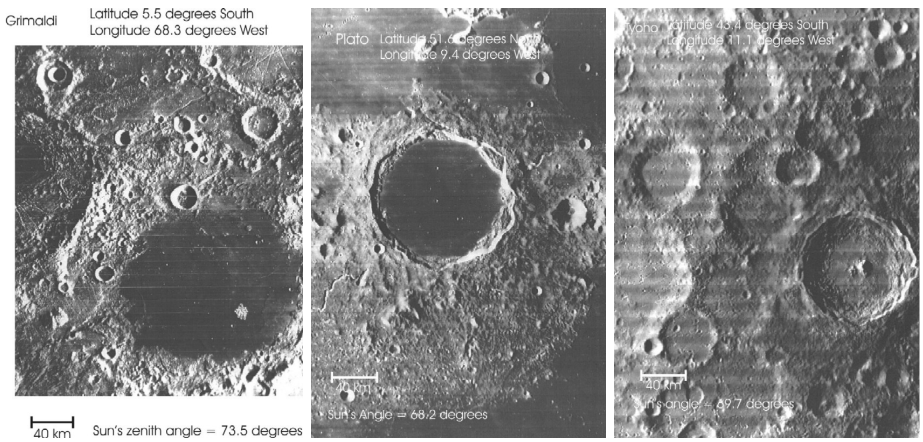

Figure (a): Photos of the Moon's surface
(from left to right: Grimaldi, Plato, Tycho)

Using the AstroArt program,[8] the diameters of craters d and shadow lengths s can be measured in pixels using the distance formula

$$\begin{aligned} d_p &\equiv \text{distance in pixels} \\ &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}, \end{aligned} \tag{1}$$

and converted to kilometres using the provided scale, as

$$\begin{aligned} d_k &\equiv \text{distance in kilometres} \\ &= \frac{d_p}{\text{scale}}. \end{aligned} \tag{2}$$

[8]F. Cavicchio, M. Nicoloni, 2020

Using the provided zenith angle θ, the height h of each crater wall can be calculated as follows:

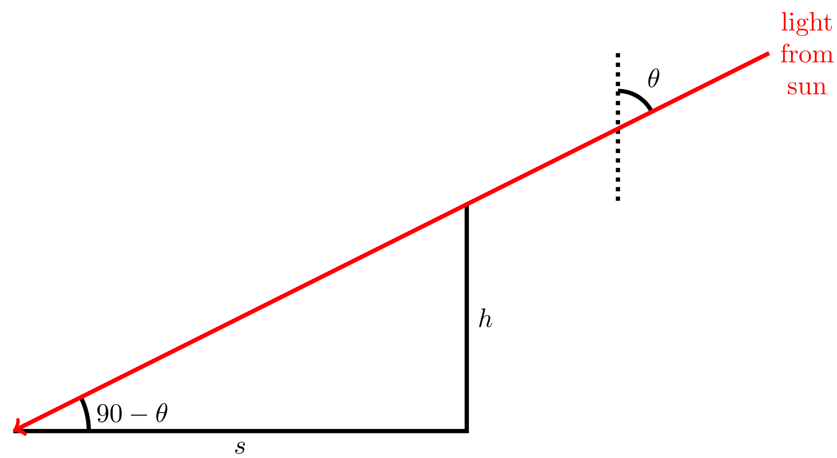

Figure (b): Diagram of crater wall and shadow

$$\begin{aligned} \tan(90-\theta) &= \frac{h}{s} \\ \implies h &= s\cot\theta. \end{aligned} \tag{3}$$

This experiment also involves calculating the kinetic energy E and mass m of each impacting body, using the following equations.

$$d = 2.5\left(\frac{E}{\rho\, g_{_M}}\right)^{\frac{1}{4}} \tag{4}$$

$$\implies E = \frac{d^4 \rho\, g_{_M}}{2.5^4} \tag{5}$$

$$= \frac{mv^2}{2}, \quad v \equiv \text{impact velocity}$$

$$\implies m = \frac{2E}{v^2} \tag{6}$$

Since bodies move at approximately 50 km s^{-1} under the influence of the Sun's gravity in the vicinity of the Earth and Moon, it follows that the range of velocities of impacting bodies v is approximately 0-100 km s^{-1} relative to the Moon. Impacting bodies that have a relative velocity of $<$ 10 km s^{-1} would not create much of an impact on the Moon's surface, and thus would result in a very small crater. It can be assumed that the craters visible in the provided images were a result of impacting bodies travelling at 10-100 km s^{-1}.

3 Equations

$$d_p = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \tag{1}$$

$$d_k = \frac{d_p}{\text{scale}} \tag{2}$$

$$h = s \cot \theta \tag{3}$$

$$d = 2.5 \left(\frac{E}{\rho\, g_{_M}} \right)^{\frac{1}{4}} \tag{4}$$

$$E = \frac{d^4 \rho\, g_{_M}}{2.5^4} \tag{5}$$

$$m = \frac{2E}{v^2} \tag{6}$$

4 Experimental Method

1. For each supplied image of the Moon, select a few craters and measure the corresponding diameters d and lengths of shadows produced s in pixels from their corresponding pixel coordinates.

2. Using the provided scale in each image, convert these measurements into km.

3. Calculate the height of each crater wall h using equation (3).

4. Plot the calculated values for h against d on a log-log plot.

5. Using equation (5), calculate the kinetic energy E of the impact that caused each crater.

6. Estimate the range of masses m of the impacting bodies using equation (6), assuming the impact velocity v is in the range of 10-100 km s^{-1}.

7. Count the total number of craters in the supplied images, and record how many are > 2 km, > 4 km, > 8 km and > 16 km.

8. By calculating the total surface area of the three images and using the data obtained in step 6, estimate the total number of craters of these various sizes on the Moon's surface.

9. Count the number of craters in the maria region and in the highland region for each image. Calculate the ratio of maria region craters to highland region craters.

5 Results

The following data was obtained from the images of Grimaldi, Plato and Tycho. The corresponding zenith angles for these images θ were 73.5°, 68.2° and 69.7°, respectively. The crater wall height was calculated using equation (3).

Table (i): Measured values of d and s and calculated values of h

Crater Diameter d, in km	Shadow length s, in km	Crater Wall Height h, in km
9.6 ± 0.8	4.4 ± 0.7	1.3 ± 0.2
10.4 ± 0.8	5.6 ± 0.7	1.7 ± 0.2
14.8 ± 0.8	8.4 ± 0.8	2.5 ± 0.3
16.0 ± 0.8	7.4 ± 0.8	2.2 ± 0.3
16.3 ± 0.8	10.4 ± 0.8	3.1 ± 0.3
17.8 ± 0.8	8.4 ± 0.8	2.5 ± 0.3
24.7 ± 0.9	11.9 ± 0.8	3.5 ± 0.3
32.6 ± 0.9	17.3 ± 0.8	5.1 ± 0.3
3.9 ± 0.7	1.8 ± 0.7	0.7 ± 0.3
5.3 ± 0.7	2.8 ± 0.7	1.1 ± 0.3
6.7 ± 0.7	3.2 ± 0.7	1.3 ± 0.3
8.3 ± 0.7	4.1 ± 0.7	1.6 ± 0.3
8.5 ± 0.7	4.6 ± 0.7	1.8 ± 0.3
8.5 ± 0.7	5.1 ± 0.7	2.0 ± 0.3
9.2 ± 0.7	5.1 ± 0.7	2.0 ± 0.3
9.4 ± 0.7	5.1 ± 0.7	2.0 ± 0.3
9.9 ± 0.7	5.2 ± 0.7	2.1 ± 0.3
11.5 ± 0.7	6.0 ± 0.7	2.4 ± 0.3
12.9 ± 0.7	6.0 ± 0.7	2.4 ± 0.3
3.5 ± 0.7	1.5 ± 0.7	0.6 ± 0.3
5.3 ± 0.7	2.1 ± 0.7	0.8 ± 0.3
5.3 ± 0.7	2.8 ± 0.7	1.0 ± 0.3
7.2 ± 0.7	3.3 ± 0.7	1.2 ± 0.3
8.8 ± 0.7	4.7 ± 0.7	1.7 ± 0.3
10.7 ± 0.7	4.9 ± 0.7	1.8 ± 0.3
13.0 ± 0.7	6.6 ± 0.7	2.4 ± 0.3
13.0 ± 0.7	7.0 ± 0.7	2.6 ± 0.3
13.5 ± 0.7	6.5 ± 0.7	2.4 ± 0.3
16.0 ± 0.7	7.4 ± 0.7	2.7 ± 0.3
20.9 ± 0.8	10.3 ± 0.7	3.8 ± 0.3
$26.5 \pm 0.8^{*}$	12.1 ± 0.7	4.5 ± 0.3

The following graph of $\log_{10} h$ against $\log_{10} d$ was plotted from the above data.

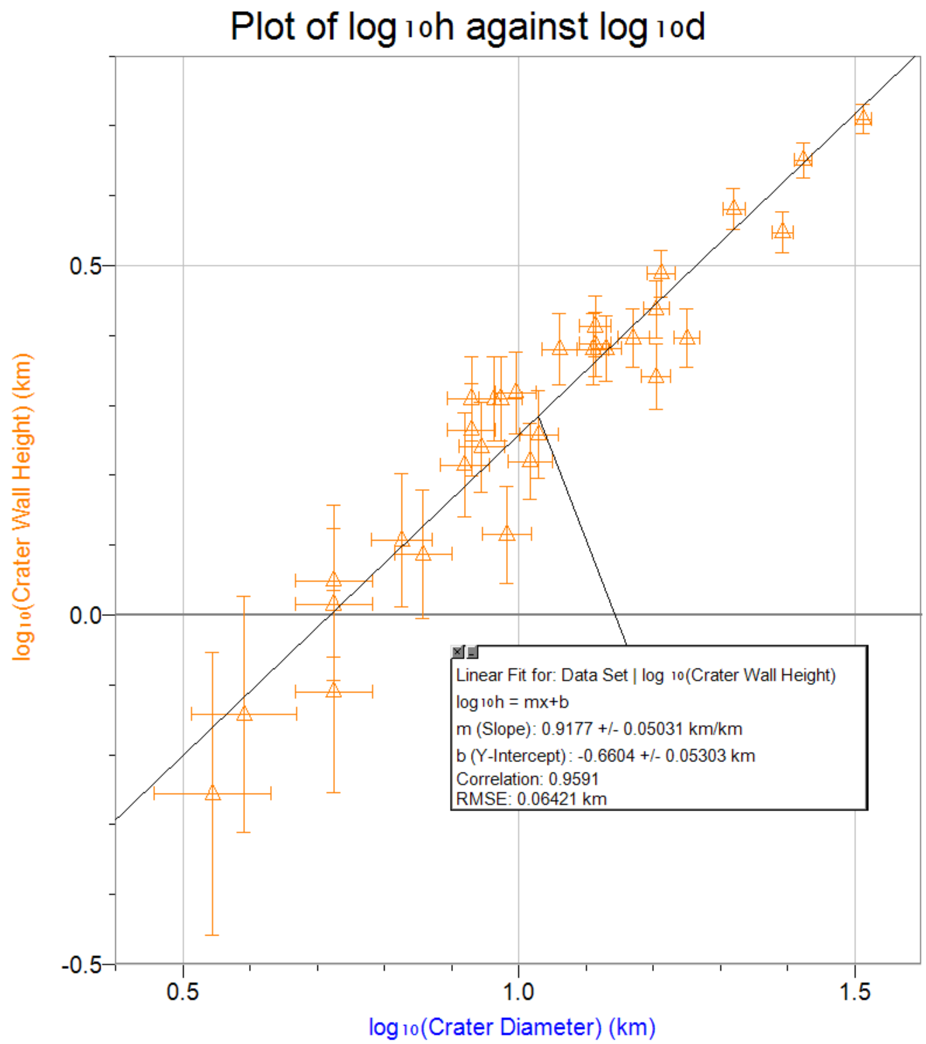

Figure (c): Plot of $\log_{10} h$ against $\log_{10} d$

Using equations (5) and (6), the kinetic energy E of each impact and range of masses $m_{\min} - m_{\max}$ of each impacting body was calculated.

Table (ii): Measured values of d and calculated values of E and m

Crater Diameter d, in m	Kinetic Energy E, in J	Mass range $m_{\min} - m_{\max}$, in kg
$9,600 \pm 800$	$(7.0 \pm 2.3) \times 10^{17}$	$(1.4 \pm 0.5) \times 10^{8} - (1.4 \pm 0.5) \times 10^{10}$
$10,400 \pm 800$	$(9.7 \pm 3.0) \times 10^{17}$	$(1.9 \pm 0.6) \times 10^{8} - (1.9 \pm 0.6) \times 10^{10}$
$14,800 \pm 800$	$(4.0 \pm 0.9) \times 10^{18}$	$(8.0 \pm 1.7) \times 10^{8} - (8.0 \pm 1.7) \times 10^{10}$
$16,000 \pm 800$	$(5.4 \pm 1.1) \times 10^{18}$	$(1.1 \pm 0.2) \times 10^{9} - (1.1 \pm 0.2) \times 10^{11}$
$16,300 \pm 800$	$(5.9 \pm 1.1) \times 10^{18}$	$(1.2 \pm 0.2) \times 10^{9} - (1.2 \pm 0.2) \times 10^{11}$
$17,800 \pm 800$	$(8.3 \pm 1.5) \times 10^{18}$	$(1.7 \pm 0.3) \times 10^{9} - (1.7 \pm 0.3) \times 10^{11}$
$24,700 \pm 900$	$(3.1 \pm 0.5) \times 10^{19}$	$(6.2 \pm 0.9) \times 10^{9} - (6.2 \pm 0.9) \times 10^{11}$
$32,600 \pm 900$	$(9.4 \pm 1.0) \times 10^{19}$	$(1.9 \pm 0.2) \times 10^{10} - (1.9 \pm 0.2) \times 10^{12}$
$3,900 \pm 700$	$(1.9 \pm 1.4) \times 10^{16}$	$(3.8 \pm 2.8) \times 10^{6} - (3.8 \pm 2.8) \times 10^{8}$
$5,300 \pm 700$	$(6.5 \pm 3.5) \times 10^{16}$	$(1.3 \pm 0.7) \times 10^{7} - (1.3 \pm 0.7) \times 10^{9}$
$6,700 \pm 700$	$(1.7 \pm 0.7) \times 10^{17}$	$(3.3 \pm 1.4) \times 10^{7} - (3.3 \pm 1.4) \times 10^{9}$
$8,300 \pm 700$	$(3.9 \pm 1.3) \times 10^{17}$	$(7.9 \pm 2.7) \times 10^{7} - (7.9 \pm 2.7) \times 10^{8}$
$8,500 \pm 700$	$(4.3 \pm 1.4) \times 10^{17}$	$(8.7 \pm 2.9) \times 10^{7} - (8.7 \pm 2.9) \times 10^{9}$
$8,500 \pm 700$	$(4.3 \pm 1.4) \times 10^{17}$	$(8.7 \pm 2.9) \times 10^{7} - (8.7 \pm 2.9) \times 10^{9}$
$9,200 \pm 700$	$(5.9 \pm 1.8) \times 10^{17}$	$(1.2 \pm 0.4) \times 10^{8} - (1.2 \pm 0.4) \times 10^{10}$
$9,400 \pm 700$	$(6.5 \pm 1.9) \times 10^{17}$	$(1.3 \pm 0.4) \times 10^{8} - (1.3 \pm 0.4) \times 10^{10}$
$9,900 \pm 700$	$(8.0 \pm 2.3) \times 10^{17}$	$(1.6 \pm 0.5) \times 10^{8} - (1.6 \pm 0.5) \times 10^{10}$
$11,500 \pm 700$	$(1.5 \pm 0.4) \times 10^{18}$	$(2.9 \pm 0.7) \times 10^{8} - (2.9 \pm 0.7) \times 10^{10}$
$12,900 \pm 700$	$(2.3 \pm 0.5) \times 10^{18}$	$(4.6 \pm 1.0) \times 10^{8} - (4.6 \pm 1.0) \times 10^{10}$
$3,500 \pm 700$	$(1.2 \pm 1.0) \times 10^{16}$	$(2.5 \pm 2.0) \times 10^{6} - (2.5 \pm 2.0) \times 10^{8}$
$5,300 \pm 700$	$(6.5 \pm 3.5) \times 10^{16}$	$(1.3 \pm 0.7) \times 10^{7} - (1.3 \pm 0.7) \times 10^{9}$
$5,300 \pm 700$	$(6.5 \pm 3.5) \times 10^{16}$	$(1.3 \pm 0.7) \times 10^{7} - (1.3 \pm 0.7) \times 10^{9}$
$7,200 \pm 700$	$(2.2 \pm 0.9) \times 10^{17}$	$(4.5 \pm 1.7) \times 10^{7} - (4.5 \pm 1.7) \times 10^{9}$
$8,800 \pm 700$	$(5.0 \pm 1.6) \times 10^{17}$	$(9.9 \pm 3.2) \times 10^{7} - (9.9 \pm 3.2) \times 10^{9}$
$10,700 \pm 700$	$(1.1 \pm 0.3) \times 10^{18}$	$(2.2 \pm 0.6) \times 10^{8} - (2.2 \pm 0.6) \times 10^{10}$
$13,000 \pm 700$	$(2.4 \pm 0.5) \times 10^{18}$	$(4.7 \pm 1.0) \times 10^{8} - (4.7 \pm 1.0) \times 10^{10}$
$13,000 \pm 700$	$(2.4 \pm 0.5) \times 10^{18}$	$(4.7 \pm 1.0) \times 10^{8} - (4.7 \pm 1.0) \times 10^{10}$
$13,500 \pm 700$	$(2.8 \pm 0.6) \times 10^{18}$	$(5.5 \pm 1.1) \times 10^{8} - (5.5 \pm 1.1) \times 10^{10}$
$16,000 \pm 800$	$(5.4 \pm 1.1) \times 10^{18}$	$(1.1 \pm 0.2) \times 10^{9} - (1.1 \pm 0.2) \times 10^{11}$
$20,900 \pm 800$	$(1.6 \pm 0.3) \times 10^{19}$	$(3.2 \pm 0.5) \times 10^{9} - (3.2 \pm 0.5) \times 10^{11}$
$26,500 \pm 800$	$(4.1 \pm 0.5) \times 10^{19}$	$(8.2 \pm 1.0) \times 10^{9} - (8.2 \pm 1.0) \times 10^{11}$

The following data of the various crater sizes was obtained from the images.

Table (iii): Number of different sized craters counted

	$2 - 4$ km	$4 - 8$ km	$8 - 16$ km	> 16 km	Total
Grimaldi	8	12	11	11	42
Plato	8	10	15	2	35
Tycho	13	22	17	15	67
Total	29	44	43	28	144

The total area of the provided images was calculated to be 275,088 km^2. Using the fact that the surface area of the moon is 38,000,000 km^2,[9] the following estimates of the number of craters of different sizes was calculated.

Table (iv): Estimated number of different sized craters on the Moon

> 2 km	> 4 km	> 8 km	> 16 km
19892	15886	9808	3868

The following graphs of number of craters against crater size and ln(number of craters) against crater size were plotted from the above data.

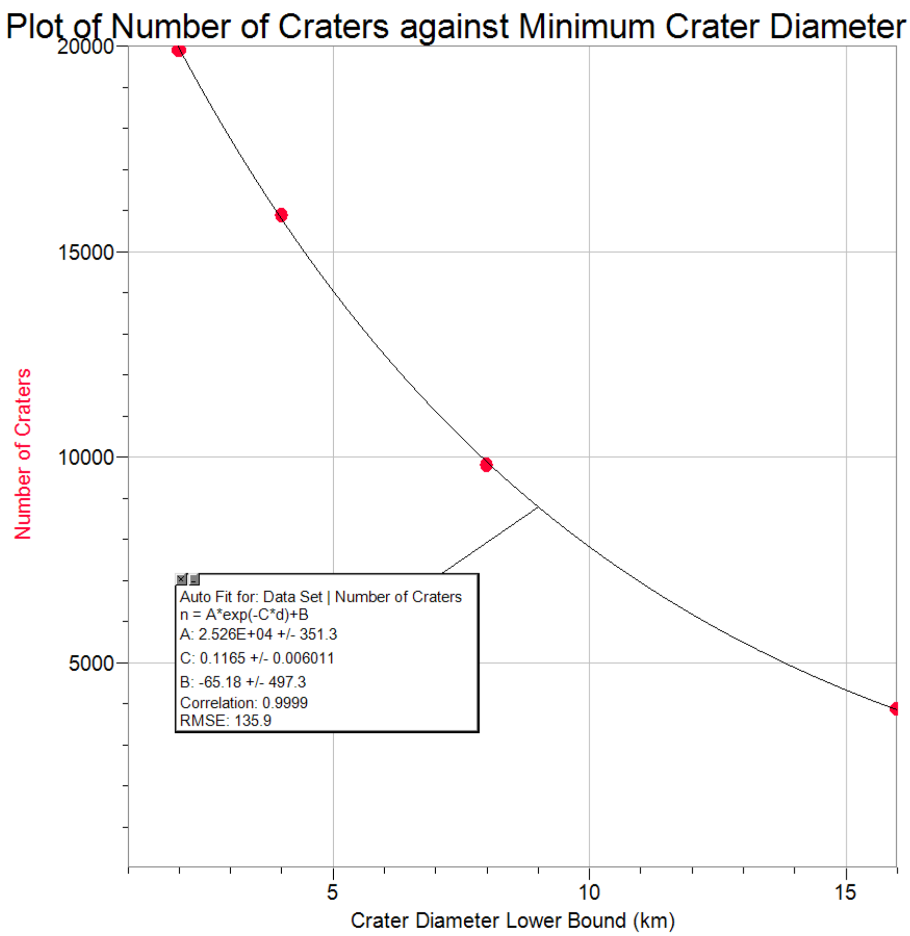

Figure (d): Plot of number of craters against minumum crater diameter

[9]Space.com, December 2020

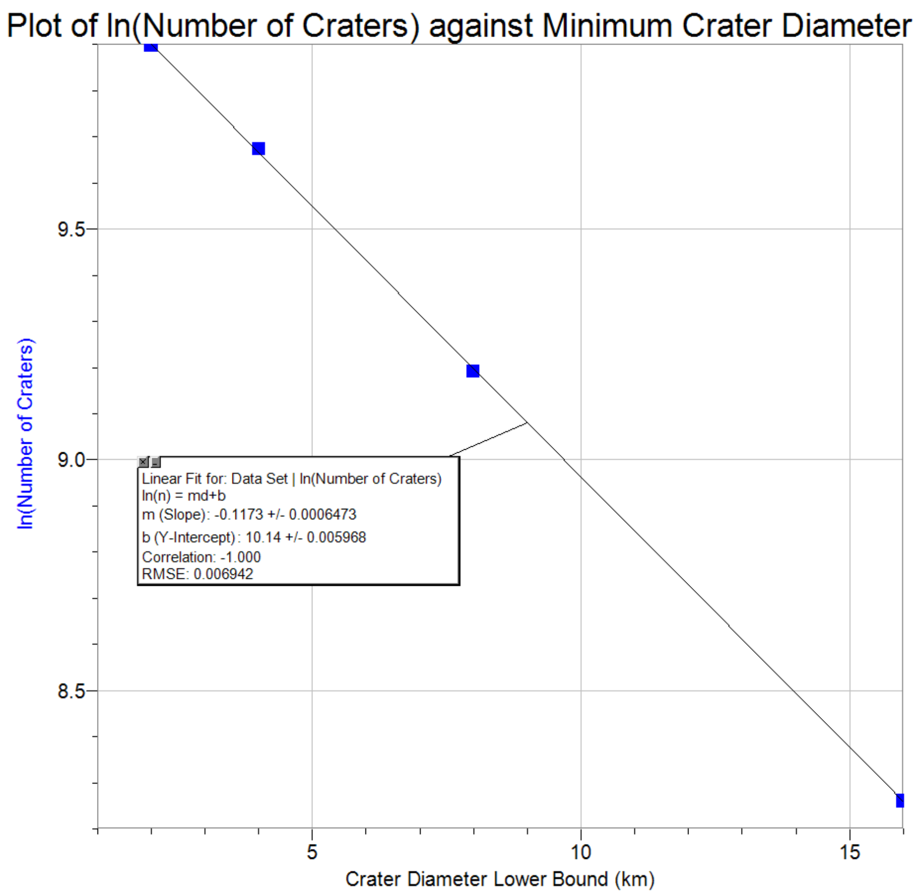

Figure (e): Plot of ln(Number of Craters) against Minimum Crater Diameter

The following data of the number of craters in the maria regions and highland regions was obtained from the images.

Table (v): Number of craters in maria regions and highland regions

	Maria region	Highland region	Total
Grimaldi	9	33	42
Plato	8	27	35
Tycho	14	53	67
Total	31	113	144

The crater ratio was thus found to be 31:113, or approximately 2:7.

6 Discussion

The provided images were not taken at full moon, as the zenith angle is not $0°$. If the images were taken at full moon, the Sun would be perpendicular to the surface of the Moon, and no shadows would have formed.

The correlation in Figure (c) is 0.9591. This suggests that there is a linear relationship between the height of the crater wall and the diameter of the crater.

If it is assumed that a crater is a square-based cylinder with base area d^2 and height h, then in order for an impacting body to create this crater, a volume of d^2h of material must be moved a distance h. Thus, the work done by the impacting body E is proportional to d^2h^2. Since it has been shown that h is proportional to d, E is thus proportional to d^4, i.e. d is proportional to $E^{\frac{1}{4}}$ This backs up the assumed equation (4).

Equation (4) can also be backed up using dimensional analysis.

$$\begin{aligned}
d &= 2.5\left(\frac{E}{\rho\, g_{_M}}\right)^{\frac{1}{4}} \\
\mathrm{m} &= \left(\frac{\mathrm{J}}{\mathrm{kg\ m^{-3}} \times \mathrm{m\ s^{-2}}}\right)^{\frac{1}{4}} \\
&= \left(\frac{\mathrm{kg\ m^{2}\ s^{-2}}}{\mathrm{kg\ m^{-2}\ s^{-2}}}\right)^{\frac{1}{4}} \\
&= \mathrm{m}
\end{aligned}$$

The range of masses of the impacting bodies varied greatly across the three images, ranging from 2.5×10^6 kg to 1.9×10^{12}.

From Figures (d) and (e), the relationship between minimum crater size and number of craters on the Moon's surface is one of exponential decay. This suggests that the majority of craters on the Moon are < 16 km and are caused by impacts of smaller bodies.

The ratio of craters in the maria regions to craters in the highland regions was found to be approximately 2:7. This lines up with the fact that craters in the maria regions are far newer than those in the highland regions, as there are far fewer craters in the maria regions than in the highland regions.

7 Error Analysis

The uncertainty in the coordinates of each pixel was taken to be a constant ± 1 pixel.

The uncertainty in the provided zenith angles was taken to be a constant $\pm 0.1°$, or $\pm \frac{\pi}{1800}$ radians.

All other uncertainties were calculated using the formula

$$\Delta f = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} \Delta x_i \right)^2}$$
$$\text{where } f = f(x_1, x_2, \ldots, x_n)$$
$$\Delta x_i \equiv \text{uncertainty in } x_i$$

*For example, the uncertainty in the crater wall height h was calculated as follows:

$$\begin{aligned}
(x_1, y_1) &= (481 \pm 1, 711 \pm 1) \\
(x_2, y_2) &= (538 \pm 1, 711 \pm 1) \\
\text{scale} &= 86 \pm \sqrt{2} \text{ pixels/40km} \\
&= 2.15 \pm \frac{\sqrt{2}}{40} \text{ pixels km}^{-1} \\
d_p &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \qquad (1) \\
&= 57 \text{ pixels} \\
\Delta d_p &= \sqrt{\left(\frac{\partial d_p}{\partial x_1} \Delta x_1 \right)^2 + \ldots + \left(\frac{\partial d_p}{\partial y_2} \Delta y_2 \right)^2} \\
&= \sqrt{2 \left(\frac{x_1 - x_2}{d} \right)^2 + 2 \left(\frac{y_1 - y_2}{d} \right)^2} \\
&= \sqrt{2} \; \forall \; x_1, x_2, y_1, y_2 \\
d_k &= \frac{d_p}{\text{scale}} \qquad (2) \\
&\approx 26.5 \text{ km} \\
\Delta d_k &= \sqrt{\left(\frac{\partial d_k}{\partial d_p} \Delta d_p \right)^2 + \left(\frac{\partial d_k}{\partial \text{scale}} \Delta \text{scale} \right)^2} \\
&= \sqrt{\left(\frac{1}{\text{scale}} \Delta d_p \right)^2 + \left(\frac{d_p}{\text{scale}^2} \Delta \text{scale} \right)^2} \\
&= 0.8 \text{ km} \\
\implies d_k &\approx 26.5 \pm 0.8 \text{ km}
\end{aligned}$$

8 Conclusion

The relationship between crater diameter and crater height is linear. Although the majority of craters are <16 km wide, the masses of the impacting bodies has a very large range. Most of the current craters on the Moon's surface are in the highland regions due to the fact that the maria have been experiencing volcanic activity which cover up craters.

9 References

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8: F. Cavicchio, M. Nicoloni, AstroArt 7, MSB Software, Ravenna, 2020

9: Space.com, `https://www.space.com/18135-how-big-is-the-moon.html`, (accessed December 2020).

The images in Figure (a) were provided by TCD School of Physics for the experiment. All other figures are of my own making, using either TikZ or LoggerPro.