Experimental Lab Report - The Fresnel Biprism

Ruaidhrí Campion
19333850
SF Theoretical Physics

October 2020

Contents

1 Abstract

In this experiment, the wavelength of light from a sodium lamp and the wavelength of light from a neon lamp were calculated using the Fresnel Biprism method. This method is very similar to, but more accurate than, Young's Slits method, where a biprism is used in place of the pair of slits to counteract the finite width of the slits used in Young's experiment.

The sodium wavelength and neon wavelength were found to be 610 ± 20 nm and 600 ± 10 nm respectively which are, within experimental error, the accepted values of 590 nm and 609 nm respectively.

2 Introduction

2.1 Young's Double Slit Experiment

Up until 1801, when Thomas Young first conducted this experiment, Isaac Newton's corpuscular theory of light, that stated light behaved as a particle, was widely accepted.[1] Young's Double Slit Experiment, however, demonstrated the wave nature of light, which later led to the concept of wave-particle duality in quantum mechanics.[2] It is for this reason that this experiment was key to our understanding of light and its behaviour.

Young's Double Slit Experiment is based on the constructive and destructive interference of waves. When two waves from coherent sources meet and are in phase, constructive interference occurs and a new wave with a larger amplitude is produced. If the waves are out of phase, then destructive interference occurs and a new wave with a smaller amplitude is produced.[2]

[1] Aspect, 2017
[2] Feynman, 1965

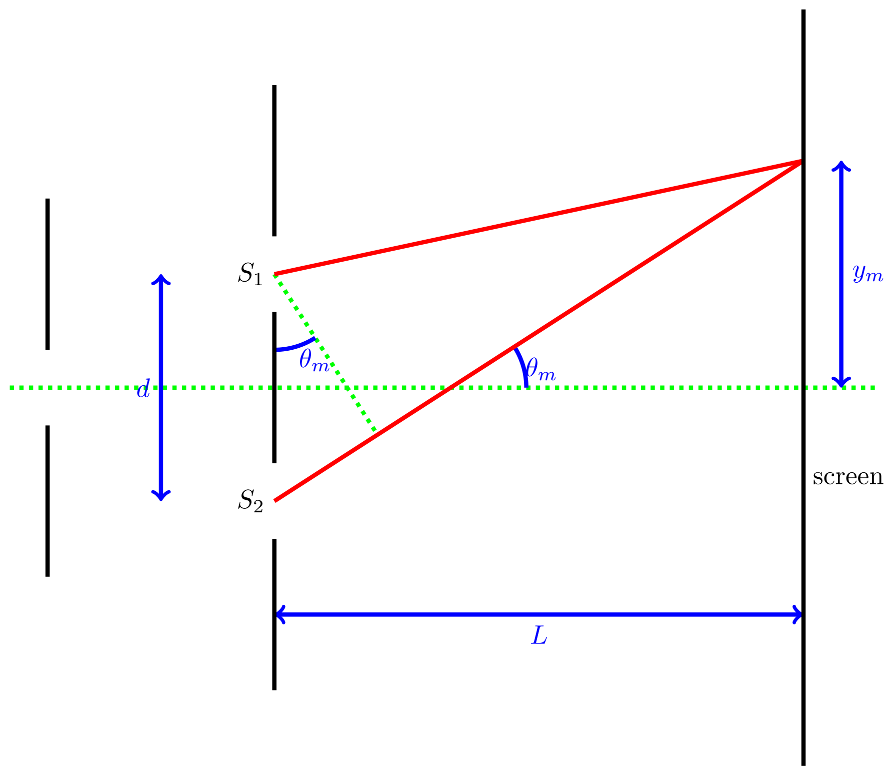

Figure 1: Young's Double Slit Experiment

$$d \equiv \text{distance between slits}$$
$$y_m \equiv \text{distance from centre to fringe}$$
$$L \equiv \text{distance from slits to screen}$$

In Young's experiment, a single light source is passed through a slit. Diffraction occurs here, radiating the light waves from the slit. These waves are then passed through a pair of slits, where diffraction again occurs. A pattern is formed on a screen, showing bright fringes where constructive interference occurs, and no light where destructive interference occurs.

Assuming that $L >> d$, the distance between successive fringes can be written as

$$\begin{aligned} S &= y_{m+1} - y_m \\ &= \frac{\lambda L(m+1)}{d} - \frac{\lambda L m}{d} \\ &= \frac{\lambda L}{d} \end{aligned}$$

The wavelength of the light can thus be expressed as

$$\lambda = \frac{Sd}{L}$$

2.2 The Fresnel Biprism Experiment

The Fresnel Biprism Experiment is an altered version of Young's Double Slit Experiment, where a Fresnel Biprism is used in place of the pair of slits.[3] This difference makes calculations much more accurate. In Young's experiment, the pair of slits are assumed to be point slits, which is not true in reality. Using the Fresnel Biprism in place of the pair of slits, the light passing through the biprism can be treated as coming from a pair of virtual point slits.

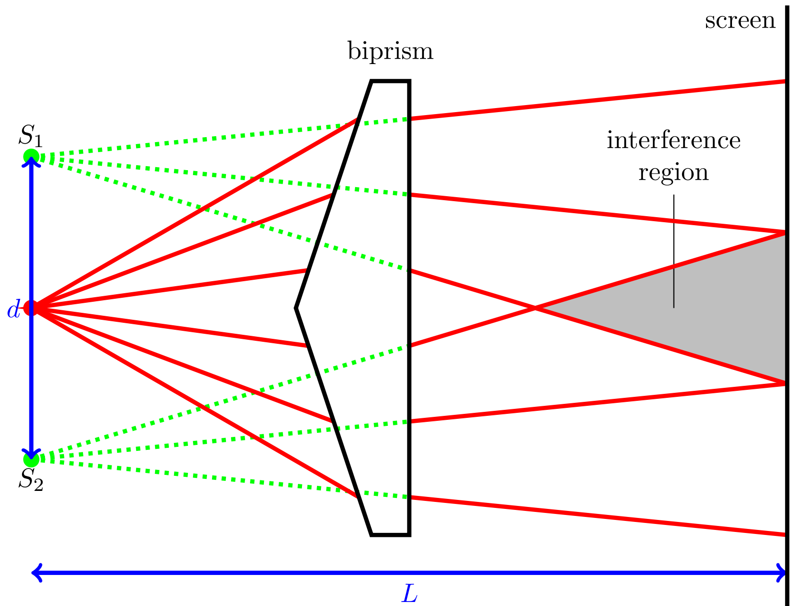

Figure 2: The Fresnel Biprism Experiment

The distance between these virtual slits cannot be measured because the slits do not physically exist. The distance between the slits can be calculated by placing a converging lens between the biprism and the screen, forming real images on the screen, and using the magnification formula

$$\frac{d_1}{d} = \frac{v_1}{u_1} \qquad \frac{d_2}{d} = \frac{v_2}{u_2}$$
$$v_2 = u_1 \qquad u_2 = v_1$$

$$\begin{aligned} \implies \frac{d_1}{d} &= \frac{u_2}{v_2} \\ &= \frac{d}{d_2} \\ \implies d &= \sqrt{d_1 d_2} \end{aligned}$$

[3]Darrigol, 2012

The wavelength can thus be expressed as

$$\lambda = \frac{S\sqrt{d_1 d_2}}{L}$$

3 Experimental Method

1. Plug in and turn on the sodium lamp to allow it to heat up until a bright light is emitted.

2. Set up the slit, biprism, lens and eyepiece on the optical bench such that they are all at the same height and in line.

3. Open the slit so that the spacing is quite narrow. Move the biprism to about 15cm from the slit and view the slit through the biprism. Rotate the biprism until closely spaces fringes can be seen.

4. Bring the eyepiece close to the biprism and look through the eyepiece. Adjust the width of the slit until the fringes can be clearly seen.

5. Bring the eyepiece to about 70cm from the slit. Place the lens between the biprism and eyepiece and ensure that a real image is formed at the eyepiece for two locations of the lens.

6. Remove the lens and calculate S by measuring the distance over about 20 fringes using the micrometer scale on the eyepiece.

7. Place the lens between the biprism and eyepiece again and locate the two locations of the lens where a real image forms at the eyepiece. Measure the distances d_1 and d_2 between the fringes for each real image using the micrometer scale on the eyepiece.

8. Using the obtained values of L, d_1, d_2 and S, calculate the wavelength of the sodium lamp λ.

9. Remove the lens again and, on the eyepiece, record the distance from a specific fringe, i.e. an arbitrary 0, to a number of consecutive fringes. Use linear regression to calculate a more accurate value of S, and again calculate λ.

10. Repeat steps 1-9, but use the neon lamp instead of the sodium lamp.

4 Results

4.1 Sodium Lamp

The distance between the slit and the eyepiece L was measured to be 0.762 ± 0.002 m.

The distances between the fringes for the real images d_1 and d_2 were measured to be 2.43 ± 0.02 mm and 1.18 ± 0.02 mm. $\sqrt{d_1 d_2}$ was calculated to be $(1.69 \pm 0.02) \times 10^{-3}$ m.

16 clear fringes and 4 possible fringes were counted across a range of 4.42 ± 0.02 mm. The number of fringes was taken to be 18 ± 2 which led to a calculated value for S of $(2.5 \pm 0.3) \times 10^{-4}$ m.

Using this data, the wavelength of the sodium light λ was calculated to be 550 ± 70 nm.

For the linear regression method of calculating S, the following data was obtained:

i	Positions y_i, in mm	$\lvert y_i - y_0 \rvert$, in mm
0	5.70 ± 0.01	0
1	5.33 ± 0.01	0.37 ± 0.02
2	5.04 ± 0.01	0.66 ± 0.02
3	4.77 ± 0.01	0.93 ± 0.02
4	4.55 ± 0.01	1.15 ± 0.02
5	4.28 ± 0.01	1.42 ± 0.02
6	4.02 ± 0.01	1.68 ± 0.02
7	3.73 ± 0.01	1.97 ± 0.02
8	3.50 ± 0.01	2.20 ± 0.02
9	3.30 ± 0.01	2.40 ± 0.02
10	2.98 ± 0.01	2.72 ± 0.02

The following graph was obtained by plotting distance from arbitrary 0 against fringe number:

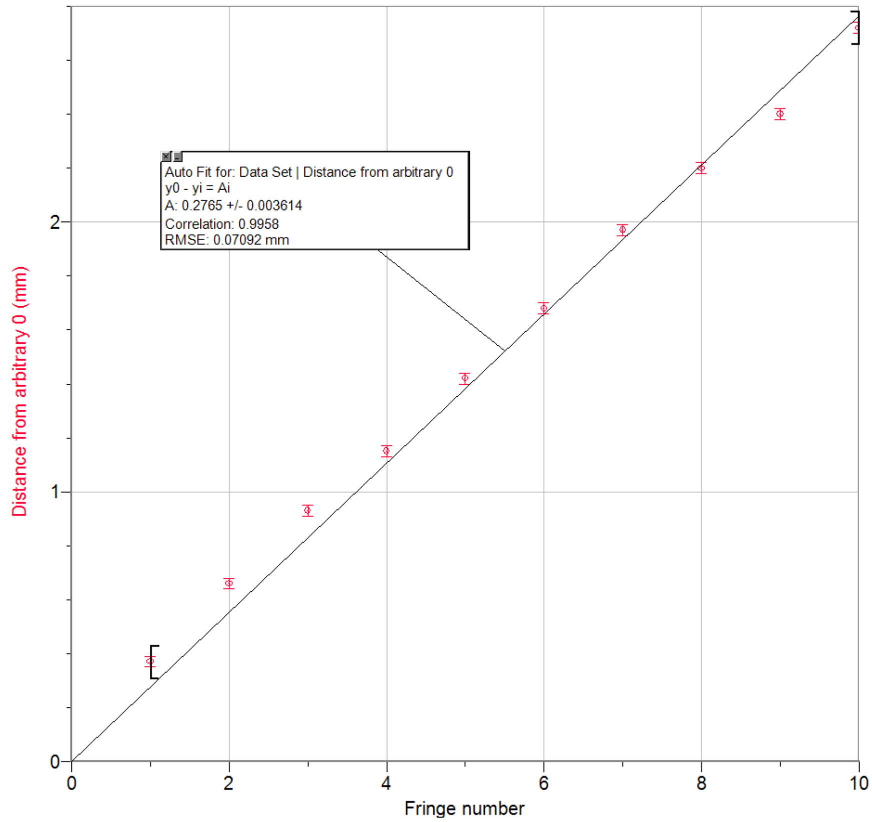

Figure 3: Linear regression to calculate S

From the graph, S was found to be $(2.77 \pm 0.04) \times 10^{-4}$ m.
Using this new value for S, λ was calculated to be 610 ± 20 nm.

4.2 Neon Lamp

The distance between the slit and the eyepiece L was measured to be 0.762 ± 0.002 m.

The distances between the fringes for the real images d_1 and d_2 were measured to be 2.05 ± 0.02 mm and 1.30 ± 0.02 mm. $\sqrt{d_1 d_2}$ was calculated to be $(1.63 \pm 0.02) \times 10^{-3}$ m.

14 clear fringes and 4 possible fringes were counted across a range of 4.40 ± 0.02 mm. The number of fringes was taken to be 16 ± 2 which led to a calculated value for S of $(2.8 \pm 0.4) \times 10^{-4}$ m.

Using this data, the wavelength of the neon light λ was calculated to be 600 ± 90 nm.

For the linear regression method of calculating S, the following data was obtained:

i	Positions y_i, in mm	$\lvert y_i - y_0 \rvert$, in mm
0	6.34 ± 0.01	0
1	6.12 ± 0.01	0.22 ± 0.02
2	5.83 ± 0.01	0.51 ± 0.02
3	5.45 ± 0.01	0.89 ± 0.02
4	5.25 ± 0.01	1.09 ± 0.02
5	4.90 ± 0.01	1.44 ± 0.02
6	4.60 ± 0.01	1.74 ± 0.02
7	4.35 ± 0.01	1.99 ± 0.02
8	4.11 ± 0.01	2.23 ± 0.02
9	3.75 ± 0.01	2.59 ± 0.02
10	3.52 ± 0.01	2.82 ± 0.02

The following graph was obtained by plotting distance from arbitrary 0 against fringe number:

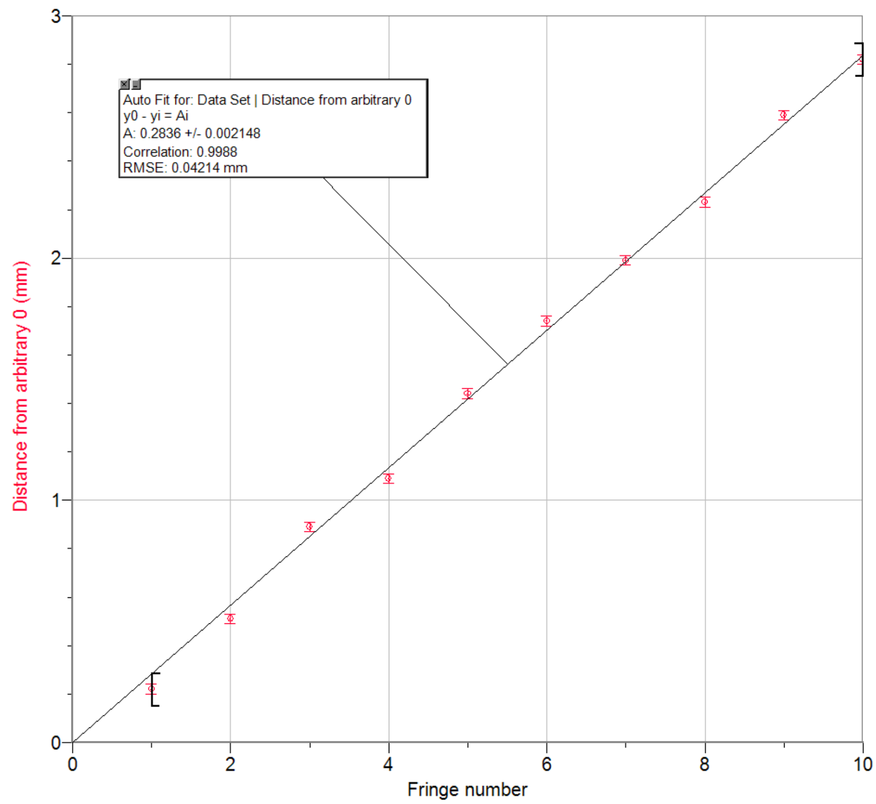

Figure 4: Linear regression to calculate S

From the graph, S was found to be $(2.84 \pm 0.03) \times 10^{-4}$ m.
Using this new value for S, λ was calculated to be 600 ± 10 nm.

5 Discussion

The values of S calculated for both the sodium lamp and neon lamp were far more accurate when calculated using the linear regression. This was expected, as for the first method, only two points were taken into account, whereas ten points were taken for the linear regression.

The calculated values for the wavelength of sodium were 550 ± 70 nm and 610 ± 20 nm, which both agree with the accepted value of 590 nm. Similarly, the calculated values for the wavelength of sodium, 600 ± 90 nm and 600 ± 10, also agree with the accepted value of 609 nm for red neon.

When looking through the eyepiece at the interference pattern, for both the sodium and neon lamps, the brightness of the fringes is not uniform. The pattern has two bright fringes on either side, with dimmer fringes within them. This is due to the fact that the biprism splits the image of the slit into two, which focuses the intensity of the light at the edges.

The sodium light actually emits two very similar wavelengths of 588.995 nm and 589.592 nm, which are commonly referred to as the sodium doublet.[4] Below is a sketch of the spectrum of the sodium doublet.

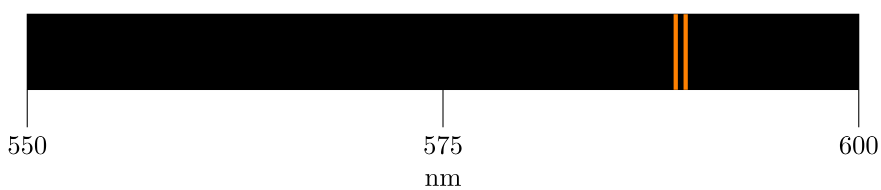

Figure 5: Sodium doublet on the wavelength spectrum

6 Error Analysis

Positions recorded using the scale on the optical bench were subject to an uncertainty of 1 mm, and positions recorded from the micrometer on the eyepiece were subject to an uncertainty of 0.01 mm.

All other uncertainties were calculated using the formula

$$\Delta f = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} \Delta x_i \right)^2}$$

$$\text{where } f = f(x_1, x_2, \ldots, x_n)$$
$$\Delta q \equiv \text{uncertainty in } q$$

For example, the uncertainty in the wavelength of the sodium light $\Delta\lambda$ (not using the linear regression method for S) was calculated as following:

[4] Nave, 2000

$$
\begin{aligned}
S &= 2.5 \times 10^{-4} \text{ m} & \sqrt{d_1 d_2} &= 1.69 \times 10^{-3} \text{ m} & L &= 0.762 \text{ m} \\
\Delta S &= 0.3 \times 10^{-4} \text{ m} & \Delta\sqrt{d_1 d_2} &= 0.02 \times 10^{-3} \text{ m} & \Delta L &= 0.002 \text{ m} \\
\frac{\partial \lambda}{\partial S} &= \frac{\sqrt{d_1 d_2}}{L} & \frac{\partial \lambda}{\partial \sqrt{d_1 d_2}} &= \frac{S}{L} & \frac{\partial \lambda}{\partial L} &= \frac{-S\sqrt{d_1 d_2}}{L^2} \\
&= \frac{169}{76200} & &= \frac{1}{3048} & &= -7.27640344 \times 10^{-7} \\
\frac{\partial \lambda}{\partial S}\Delta S &= 6.654 \times 10^{-8} \text{ m} & \frac{\partial \lambda}{\partial \sqrt{d_1 d_2}}\Delta\sqrt{d_1 d_2} &= 6.56 \times 10^{-9} \text{ m} & \frac{\partial \lambda}{\partial L}\Delta L &= -1.46 \times 10^{-9} \text{ m}
\end{aligned}
$$

$$
\begin{aligned}
\implies \Delta\lambda &= \sqrt{\left(\frac{\partial \lambda}{\partial S}\Delta S\right)^2 + \left(\frac{\partial \lambda}{\partial \sqrt{d_1 d_2}}\Delta\sqrt{d_1 d_2}\right)^2 + \left(\frac{\partial \lambda}{\partial L}\Delta L\right)^2} \\
&= \sqrt{(6.654 \times 10^{-8})^2 + (6.56 \times 10^{-9})^2 + (-1.46 \times 10^{-9})^2} \\
&= 6.688 \times 10^{-8} \text{ m} \\
&\approx 70 \text{ nm}
\end{aligned}
$$

7 Conclusion

The Fresnel Biprism experiment is a more accurate method to calculate the wavelength of a light source. By using this method, the wavelength of sodium light was calculated to be 610 ± 20 nm, and the wavelength of red neon light was calculated to be 600 ± 10 nm. The relatively large uncertainty arises from the calculation of the fringe separation.

8 References

1: Aspect, Alain (2017). "From Huygens' waves to Einstein's photons: Weird light". Comptes Rendus Physique. pp. 498-503.

2: Feynman, Richard P.; Robert B. Leighton; Matthew Sands (1965). The Feynman Lectures on Physics, Vol. 3: Quantum Mechanics. Addison-Wesley.

3: Darrigol, Olivier (2012). A History of Optics from Greek Antiquity to the Nineteenth Century. p. 205.

4: Nave, Rod (2000). HyperPhysics.
http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/sodzee.html

All figures are of my own making, either using Logger Pro or TikZ.