

Linear algebra problems (week 5)

1. A square matrix A is called upper triangular, if all the entries below its diagonal are zero, namely if $A_{ij} = 0$ whenever $i > j$. Suppose that both A and B are $n \times n$ upper triangular matrices. Show that their product AB is upper triangular as well.
2. Show that the reduced row echelon form of A is the identity matrix I_2 when

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad ad - bc \neq 0.$$

3. Letting A and I_2 be as in the previous problem, show that

$$A^2 - (a + d)A + (ad - bc)I_2 = 0.$$