Analysis and Algebra problems (week 3)

1. On $\mathbb{R}$, define $x \sim y$ means $x^2 = y^2$. Is $\sim$ an equivalence relation? If so, what are the equivalence classes? What is a "natural" set of equivalence class representatives?

2. On $\mathbb{R}$, define $x \sim y$ means $x - y \in \mathbb{Z}$. Is $\sim$ an equivalence relation? If so, what are the equivalence classes? What is a "natural" set of equivalence class representatives?

3. The angle between two planes is the angle between their normal vectors. Find the angle between the planes $x + y + 2z = 3$ and $2x - y + z = 6$.

4. Find the equation of the line through $A(1, 2, 1)$ which is perpendicular to the line

$$x = 1 + t, \qquad y = 2 - 3t, \qquad z = 2t$$

and also parallel to the plane $2x - z = 1$.

5. Find the equation of the plane which contains the point $P(1, -2, 5)$ and the line

$$x = 1 + t, \qquad y = 2 + 2t, \qquad z = 3 + 2t.$$