Linear algebra problems (week 12)

1. Find the matrix of $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ with respect to the bases B_1 and B_2 when

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x+y \\ 2x \\ y \end{bmatrix}, \qquad B_1 = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}, \qquad B_2 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

2. Find the linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ whose matrix with respect to the bases of the previous problem is the matrix

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}.$$