Linear algebra II
Tutorial problems #9

1. Let P_1 be the space of all real polynomials of degree at most 1 and define

$$\langle f, g\rangle = \int_0^1 (ax+1)f(x)g(x)\,dx$$

for all $f, g \in P_1$. Find the values of a for which the signature of this form is $(1, 1)$.

2. Suppose that A is a square matrix such that Q^tAQ is diagonal for some orthogonal matrix Q. Show that A is necessarily symmetric.

3. Show that $I + A^tA$ is invertible for every square matrix A. Hint: if it is not invertible, then -1 is an eigenvalue of A^tA.