Linear algebra I
Tutorial problems #8

1. Show that the vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n$ form a basis of $\mathbb{R}^n$ if and only if the matrix A whose columns are these vectors is invertible. Hint: pivots in each column/row.

2. Suppose that the vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n$ form a basis of $\mathbb{R}^n$ and let $\boldsymbol{v} \in \mathbb{R}^n$ be arbitrary. Show that there is a unique way of expressing $\boldsymbol{v}$ as a linear combination of the $\boldsymbol{v}_i$'s.

3. Express $\boldsymbol{v}$ as a linear combination of the columns of A when

$$\boldsymbol{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \qquad A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}.$$