Linear algebra II
Tutorial problems #5

1. Suppose that A is a 2×2 matrix with distinct eigenvalues $\lambda = a, b$. Show that

$$A^n = \frac{a^n - b^n}{a - b} \cdot A + \frac{ab^n - ba^n}{a - b} \cdot I,$$

where I is the identity matrix. Hint: prove the formula in the case that A is diagonal and then relate the powers of A to those of its Jordan form.

2. Compute the power A^n for any positive integer n when

$$A = \begin{bmatrix} 3 & 1 \\ -2 & 0 \end{bmatrix}.$$

3. Let P_2 be the space of all real polynomials of degree at most 2 and define

$$\langle f, g \rangle = \int_0^1 (2x - 1) f(x) g(x)\, dx$$

for all $f, g \in P_2$. Show that this is a bilinear form and find its matrix with respect to the standard basis.