Linear algebra II
Tutorial problems #3

1. The following matrices have eigenvalues $\lambda = 1, 2, 2$. Find their Jordan form.

$$A_1 = \begin{bmatrix} 0 & 7 & 1 \\ -1 & 4 & 1 \\ 0 & 3 & 1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -3 & 5 & 5 \\ -1 & 3 & 1 \\ -3 & 3 & 5 \end{bmatrix}.$$

2. Show that the trace of a square matrix is equal to the sum of its eigenvalues. Hint: prove this for the Jordan form of the matrix and then use similarity.

3. Given a square matrix A with eigenvalue λ, we know that the null spaces $\mathcal{N}(A - \lambda I)^j$ are increasing with j and eventually stabilize. Compute these null spaces when

$$A = \begin{bmatrix} \lambda & 1 & & \\ & \lambda & 1 & \\ & & \lambda & 1 \\ & & & \lambda \end{bmatrix}.$$