MA011 (25, 26 Jan 2012)

For an n-by-n matrix A, the *trace* of A, denoted $\mathrm{tr}(A)$, is the sum of the elements on the main diagonal of A.

1. Prove that for any n-by-n matrix A and for any real number α,

$$\mathrm{tr}(\alpha A) = \alpha \mathrm{tr}(A).$$

2. Prove that for any n-by-n matrices A and B,

$$\mathrm{tr}(A + B) = \mathrm{tr}(A) + \mathrm{tr}(A).$$

3. Prove that for any n-by-n matrices A and B,

$$\mathrm{tr}(AB) = \mathrm{tr}(BA).$$

4. Show that there are no n-by-n matrices A and B such that

$$AB - BA = I.$$