PDEs, Homework #1
Problems: 2, 4, 5, 6, 7
due Thursday, Oct. 29

1. Show that the linear change of variables

$$v = x + y, \qquad w = x - y$$

transforms the equation $u_{xy} = 0$ into the wave equation $u_{vv} - u_{ww} = 0$.

2. Which of the following PDEs are linear? Which of those are homogeneous?

$$x^2 u_x + y^2 u_y = \sin(xy), \qquad e^x u_x + e^u u_y = 0, \qquad x u_{xx} + y u_{yy} = u_x.$$

3. Find all solutions $u = u(x, y)$ of the equation $u_{xy} = xy$.

4. Find all separable solutions $u = F(x)G(y)$ of the equation $x u_y = y u_x$.

5. Find all separable solutions $u = F(x)G(y)H(z)$ of the equation $u_x - u_y + u_z = 0$.

6. Find all solutions $u = u(x, t)$ of the equation $u_t + 2xt u_x = e^t$.

7. Find a function $f(x)$ for which the initial value problem

$$u_x + u_y = 2xu, \qquad u(x, x) = f(x)$$

has no solutions and a function $f(x)$ for which it has infinitely many solutions.

8. Show that the characteristic curves for the equation $y u_x - x u_y = 0$ are circles around the origin. Conclude that $u(x, 0) = f(x)$ must be even for any solution u.

9. Find all solutions $u = u(x, y)$ of the equation $u_x + u_y + u = e^{y-x}$.

10. Find all solutions $u = u(x, y, z)$ of the initial value problem

$$x u_x + 2y u_y + u_z = 3u, \qquad u(x, y, 0) = f(x, y).$$

11. Which of the following second-order equations are hyperbolic? elliptic? parabolic?

$$u_{xx} - 2u_{xy} + u_{yy} = 0, \qquad 3u_{xx} + u_{xy} + u_{yy} = 0, \qquad u_{xx} - 5u_{xy} - u_{yy} = 0.$$

12. For which values of a is the equation $a u_{xx} + a u_{xy} + u_{yy} = 0$ elliptic?