# Some definitions

**Definition 1 (ODE/PDE).** A differential equation is an equation that involves a function $u$ and its derivatives. It is called ordinary or an ODE, if $u$ depends on one variable, in which case derivatives are denoted by primes $u', u''$ etc. It is called partial or a PDE, if $u$ depends on two or more variables, in which case derivatives are denoted by subscripts $u_x, u_{xy}$ etc.

**Definition 2 (Order).** The order of an ODE/PDE is defined as the order of the highest-order derivative which appears in the equation. For instance, $u' = u$ is a first-order ODE, while $u_{xx} = u_y$ is a second-order PDE.

**Definition 3 (Linear, homogeneous).** An ODE/PDE is called linear, if the coefficients of the unknown function $u$ and its derivatives are all independent of $u$. A linear ODE/PDE that has $u = 0$ as a solution is called homogeneous.

**Example 4.** The equation $u_x + x^2 u_y = e^{xy}$ is linear, whereas $u_t = uu_x$ is nonlinear.

**Example 5.** Every second-order linear ODE for the function $u = u(x)$ has the form

$$a(x)u'' + b(x)u' + c(x)u = d(x).$$

Similarly, every first-order linear PDE for the function $u = u(x, y)$ has the form

$$a(x, y)u_x + b(x, y)u_y + c(x, y)u = d(x, y).$$

In either case, to say that the equation is homogeneous is to say that $d = 0$.

**Lemma 6 (Superposition principle).** If an ODE/PDE is linear homogeneous, then

(a) the sum of any two solutions is itself a solution and

(b) every scalar multiple of a solution is also a solution.