UNIVERSITY OF DUBLIN

XMA2161

TRINITY COLLEGE

Faculty of Engineering, Mathematics
and Science

school of mathematics

SF Maths, SF TP **Trinity Term 2009**
JS TSM

Course 216

Tuesday, May 19 Luce Hall 9:30 – 11:30

Dr. P. Karageorgis

ATTEMPT FOUR QUESTIONS.
Log tables are available from the invigilators, if required.

1. *(5 points each)* Prove each of the following statements.

   (a) Every solution of $x''(t) + x(t) = 0$ is bounded.

   (b) Every solution of $x''(t) + x(t) = \sin t$ is unbounded.

   (c) Every solution of $x''(t) + x(t) = \sin(2t)$ is bounded.

   (d) The initial value problem $tx'(t) = x(t)$, $x(0) = 1$ has no solutions.

   (e) The initial value problem $tx'(t) = x(t)$, $x(0) = 0$ has infinitely many solutions.

2. *(25 points)*

   (a) *(5 points)* Find the unique solution $y = y(t)$ of the initial value problem

$$y' + 2ty = 0, \qquad y(0) = e.$$

   (b) *(20 points)* Find the unique solution $y = y(t)$ of the initial value problem

$$y'' - \frac{y'}{t} + \frac{y}{t^2} = t \log t, \qquad y(1) = y'(1) = 0.$$

   As a hint, note that the left hand side of the ODE is a perfect derivative.

3. *(25 points)*

   (a) *(10 points)* Check that $y_1(t) = 1/t$ is a solution of the second-order ODE

$$t^2 y'' + 3ty' + y = 0, \qquad t > 0$$

   and then use this fact to find all solutions of the ODE.

   (b) *(15 points)* Find all solutions $y = y(t)$ of the third-order ODE

$$y''' + y'' - 4y' - 4y = 4e^{-2t}.$$

4. *(25 points)*

   (a) *(20 points)* Find all solutions of the autonomous linear system

   $$x'(t) = x(t) - y(t), \qquad y'(t) = x(t) + y(t).$$

   (b) *(5 points)* Is the zero solution stable? Is it asymptotically stable?

5. *(25 points)*

   (a) *(5 points)* Show that the zero solution is an unstable solution of the system

   $$x'(t) = x + y - x^2 - y^2, \qquad y'(t) = 2x + y - x^2 y.$$

   (b) *(5 points)* Show that the zero solution is an asymptotically stable solution of

   $$x'(t) = -2xy^2 - x^3, \qquad y'(t) = x^2 y - y.$$

   As a hint, try to show that $V(x, y) = x^2 + y^2$ is a strict Lyapunov function.

   (c) *(15 points)* Show that the zero solution is an asymptotically stable solution of

   $$x'(t) = -2x - y^2, \qquad y'(t) = -x^2 - y.$$

   As a hint, try to show that $V(x, y) = x^2 + y^2$ is a strict Lyapunov function.