ODEs, Homework #5
Practice problems

1. In each of the following cases, is the zero solution stable? Asymptotically stable?

(a) $x' = 3x - 2y, \; y' = 2x - 2y$ (b) $x' = x - 5y, \; y' = x - 3y$ (c) $x' = -y, \; y' = 4x$

2. Show that the zero solution is a stable solution of the system

$$x' = y, \qquad y' = -y - x^3$$

by finding a Lyapunov function of the form $V(x, y) = \alpha x^4 + \beta y^2$.

3. Show that the zero solution is an asymptotically stable solution of the system

$$x' = -x - xy^2, \qquad y' = -y - x^2 y$$

by finding a strict Lyapunov function of the form $V(x, y) = \alpha x^2 + \beta y^2$.

4. Find all critical points of the system

$$x' = x(10 - x - y), \qquad y' = y(30 - 2x - y)$$

and then classify them as stable, asymptotically stable or unstable.

5. Find all critical points of the system

$$x' = y^2 - x, \qquad y' = x^2 - y$$

and then classify them as stable, asymptotically stable or unstable.