ODEs, Homework #4
First five problems:
due Friday, April 1

1. Check that $y_1(t) = t$ is a solution of the second-order ODE

$$(t\cos t - \sin t)y'' + y't\sin t - y\sin t = 0$$

and then use this fact to find all solutions of the ODE.

2. Show that the zero solution is the only bounded solution of

$$x'(t) = x + xy^2, \qquad y'(t) = y - x^2y.$$

3. For which of the following ODEs is the zero solution stable? Asymptotically stable?

(a) $x' = -2y,\ y' = 2x$ (b) $x' = x + 2y,\ y' = x$ (c) $x' = x - 5y,\ y' = 5x + y$

4. For which of the following ODEs is the zero solution stable? Asymptotically stable?

(a) $x'(t) = x(t)^2$ (b) $x'(t) = -x(t)^3$ (c) $x'(t) = x(t)\cos t$

5. Use the substitution $z = \log y(t)$ to solve the equation $y' = y(\log y - 1)$.

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6. Check that $y_1(t) = e^t$ is a solution of the second-order ODE

$$(t^2 + t)y'' - (t^2 - 2)y' - (t + 2)y = 0$$

and then use this fact to find all solutions of the ODE.

7. Check whether the zero solution is a stable or unstable solution of $\boldsymbol{y}' = A\boldsymbol{y}$ when

$$A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}, \qquad A = \begin{bmatrix} 1 & -4 \\ 4 & -7 \end{bmatrix}, \qquad A = \begin{bmatrix} 1 & -5 \\ 1 & -3 \end{bmatrix}.$$

8. Use the substitution $w = 1/y(t)$ to solve the equation $ty' + y = y^2\log t$.

9. Show that the zero solution is an unstable solution of the system

$$x'(t) = x + 2y + xy, \qquad y'(t) = y - 2x - x^2.$$

Hint: find and solve the ODE satisfied by $E(t) = x(t)^2 + y(t)^2$.

10. Let $a \in \mathbb{R}$ and consider the second-order equation

$$y''(t) + 2ay'(t) + y(t) = 0.$$

For which values of a is the zero solution stable? Asymptotically stable?