ODEs, Homework #3
First five problems:
due Friday, March 11

1. Suppose A, B are constant square matrices such that $e^{tA}e^{tB} = e^{t(A+B)}$ for all $t \in \mathbb{R}$. Show that $AB = BA$. Hint: differentiate twice and let $t = 0$.

2. Compute the matrix exponential e^{tA} in the case that

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}.$$

3. Compute the matrix exponential e^{tA} in the case that $A = \begin{bmatrix} 1 & 2 \\ -4 & -3 \end{bmatrix}$.

4. Let $x_0, v_0 \in \mathbb{R}$ be fixed. Find the unique solution of the initial value problem

$$x''(t) - 2x'(t) + 2x(t) = e^t, \qquad x(0) = x_0, \qquad x'(0) = v_0.$$

5. Find all solutions of the non-homogeneous scalar ODE

$$x''(t) - 2x'(t) + 2x(t) = te^{2t}.$$

. . . .

6. Find all solutions of the non-homogeneous third-order scalar ODE

$$y'''(t) - 2y''(t) - y'(t) + 2y(t) = \sin t.$$

7. Find all solutions of the non-homogeneous scalar ODE

$$y''(t) + 2y'(t) + y(t) = 2e^{-t} + t.$$

8. Find all solutions of the non-homogeneous scalar ODE

$$y''(t) - 3y'(t) + 2y(t) = t^2 + t + 1.$$

9. Determine the unique solution of the initial value problem

$$x'(t) = x - y, \qquad y'(t) = x + y, \qquad x(0) = x_0, \qquad y(0) = y_0.$$

10. Show that $E(t) = x(t)^2 + y(t)^2$ is decreasing for all solutions x, y of the system

$$x'(t) = -xy^3 - x, \qquad y'(t) = x^2y^2 - y.$$