ODEs, Homework #2
First five problems:
due Friday, Feb. 18

1. Find all solutions of the system of ODEs

$$x'(t) = -12x(t) - 16y(t), \qquad y'(t) = 11x(t) + 15y(t).$$

2. Let $x_0, v_0 \in \mathbb{R}$ be given. Find the unique solution of the initial value problem

$$x''(t) - 3x'(t) + 2x(t) = 0, \qquad x(0) = x_0, \qquad x'(0) = v_0.$$

3. The system $\mathbf{y}' = A\mathbf{y}$ can always be solved directly when A is upper triangular. Prove this in the case that A is 2×2 upper triangular with constant entries, say

$$\begin{bmatrix} x \\ y \end{bmatrix}' = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

for some $a, b, c \in \mathbb{R}$. Hint: solve the second equation and then solve the first.

4. Compute the exponential e^{tA} when $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.

5. Find all solutions $x = x(t)$ of the third-order equation $x''' - x'' - 4x' + 4x = 0$.

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6. Compute the exponential e^{tA} when $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.

7. Compute the exponential e^{tA} when

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}.$$

8. Find all solutions of the system of ODEs

$$x'(t) = -3x(t) + y(t), \qquad y'(t) = -7x(t) + 5y(t).$$

9. Show that every solution of $x''(t) + 4x'(t) + 3x(t) = 0$ is such that

$$\lim_{t\to\infty} x(t) = 0.$$

10. Determine the unique solution of the initial value problem

$$\boldsymbol{y}'(t) = A\boldsymbol{y}(t), \qquad \boldsymbol{y}(0) = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \qquad A = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}.$$