ODEs, Homework #1
First five problems:
due Friday, Feb. 4

1. Determine all functions $y = y(t)$ such that $y' = \frac{2ty^2}{1+t^2}$.

2. Determine all functions $y = y(x)$ such that $y' - \frac{y}{x+1} = x$.

3. Which of the following ODEs are linear? Which of those are homogeneous?

$$s^2x'(s) + s^4x(s) = s, \qquad t^2y''(t) + y(t)y'(t) = 1,$$
$$xy''(x) + \sin y(x) = 0, \qquad xy''(x) + e^xy'(x) = 0.$$

4. Find all solutions of the equation

$$y'(t) - \frac{y(t)}{t\log t} = \frac{1}{t}, \qquad t > 0.$$

5. Find the unique solution $y = y(t)$ of the initial value problem

$$y' = y(1-y)\cos t, \qquad y(0) = y_0.$$

Hint: separate variables and then use partial fractions.

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6. Let $a, y_0 \in \mathbb{R}$ and suppose that f is continuous. Solve the initial value problem

$$y'(t) - ay(t) = f(t), \qquad y(0) = y_0.$$

7. Show that the initial value problem

$$ty'(t) = y(t), \qquad y(0) = 1$$

has no solutions. Why doesn't this contradict our existence result, Theorem 3.1?

8. Solve the initial value problem

$$y'(t) - \frac{2y(t)}{t} = 4t^3, \qquad y(1) = 3.$$

9. Find all solutions of the equation

$$(x-2)y'(x) = (x-1)y(x).$$

10. Show that the unique solution of the initial value problem

$$y'(t) = \frac{2t}{1+t^2} \cdot \sin y(t), \qquad y(0) = 1$$

exists for all times. Hint: use the associated integral equation.