UNIVERSITY OF DUBLIN

XMA2161

TRINITY COLLEGE

FACULTY OF ENGINEERING, MATHEMATICS
AND SCIENCE

SCHOOL OF MATHEMATICS

SF Maths, SF TP
JS TSM

Trinity Term 2009

COURSE 216

Tuesday, May 19 Lower Luce Hall 9:30 – 11:30

Dr. P. Karageorgis

ATTEMPT FOUR QUESTIONS.
Log tables are available from the invigilators, if required.

1. *(25 points)* For each of the following equations, state whether *all* solutions are bounded for $t > 0$ and whether *all* solutions satisfy $\lim_{t\to\infty} x(t) = 0$.

   (a) *(5 points)* $x''(t) + 3x'(t) + 2x(t) = 0$

   (b) *(5 points)* $x''(t) + 2x'(t) + 3x(t) = 0$

   (c) *(5 points)* $x''(t) - 6x'(t) + 10x(t) = 0$

   (d) *(5 points)* $x''(t) + 6x'(t) + 10x(t) = 0$

   (e) *(5 points)* $x'''(t) + x''(t) + x'(t) + x(t) = 0$

2. *(25 points)*

   (a) *(10 points)* Find the unique solution $y = y(t)$ of the initial value problem

   $$y' - \frac{ty}{t^2+1} = t, \qquad y(0) = 0.$$

   (b) *(10 points)* Find the unique solution $y = y(t)$ of the initial value problem

   $$y' = 1 + 2t + y + 2ty, \qquad y(0) = 2.$$

   (c) *(5 points)* Show that $y = y(t)$ is a solution of the ODE

   $$y' = f(y/t), \qquad t > 0$$

   if and only if $z = y/t$ is a solution of the separable ODE

   $$tz' = f(z) - z, \qquad t > 0.$$

3. *(25 points)*

   (a) *(15 points)* Check that $y_1(t) = e^t$ is a solution of the second-order ODE

   $$(t-1)y'' - ty' + y = 0$$

   and then use this fact to find all solutions of the ODE.

   (b) *(10 points)* Find all solutions $y = y(t)$ of the third-order ODE

   $$y''' - y'' - y' + y = 4e^{-t}.$$

4. *(25 points)*

   (a) *(15 points)* Let $a \in \mathbb{R}$ be fixed and consider the autonomous linear system

   $$x'(t) = ax(t) + y(t), \qquad y'(t) = x(t) + ay(t).$$

   For which values of a is the zero solution stable? asymptotically stable?

   (b) *(10 points)* Let $a \in \mathbb{R}$ be fixed and consider the autonomous linear system

   $$x'(t) = -x(t) + y(t), \qquad y'(t) = x(t) - ay(t).$$

   For which values of a is $V(x,y) = x^2 + y^2$ a strict Lyapunov function?

5. *(25 points)*

   (a) *(15 points)* Find a second-order linear ODE of the form

   $$y''(t) + p(t)y'(t) + q(t)y(t) = r(t)$$

   such that each of $y_1(t) = e^t$, $y_2(t) = e^t + 2e^{-t}$ and $y_3(t) = e^t + e^{2t}$ are solutions.

   (b) *(10 points)* Suppose b, c are positive real numbers and consider the ODE

   $$y''(t) + by'(t) + cy(t) = 0.$$

   Show that every solution to this ODE is such that $\lim_{t\to\infty} y(t) = 0$.