MA121 Tutorial Problems #6

1. Test each of the following series for convergence:

$$\sum_{n=1}^{\infty} \frac{n+1}{n^2+1}\,, \qquad \sum_{n=1}^{\infty} \frac{n}{2^n}\,, \qquad \sum_{n=2}^{\infty} \frac{\log n}{n}\,, \qquad \sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}\,.$$

2. Test each of the following series for convergence:

$$\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n, \qquad \sum_{n=1}^{\infty} \frac{n+2}{n^3+1}\,, \qquad \sum_{n=1}^{\infty} \frac{n!}{n^n}\,, \qquad \sum_{n=1}^{\infty} \log\left(1+\frac{1}{n}\right).$$

3. Compute each of the following sums:

$$\sum_{n=0}^{\infty} \frac{3^{n+2}}{2^{2n+1}}\,, \qquad \sum_{n=1}^{\infty} \frac{5^{n+1}}{2^{3n}}\,.$$

- These are all practice problems, not a homework assignment.
- However, part of your next homework assignment will be based on these problems.
- In case you get stuck, some hints are provided on the other page of this sheet.

Hints and comments

1a. Use the limit comparison test to compare with $b_n = 1/n$.

1b. Use the ratio test; you should find that $L = 1/2$.

1c. Use the comparison test to compare with $b_n = (\log 2)/n$.

1d. Use the ratio test; you should find that $L = 1/4$.

2a. Use the nth term test; in this case, the nth term is approaching e.

2b. Use the limit comparison test to compare with $b_n = 1/n^2$.

2c. Use the ratio test; you will get a limit that is slightly tricky, namely

$$L = \lim_{n\to\infty} \frac{n^n}{(n+1)^n} = \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = \frac{1}{e}.$$

2d. Use the limit comparison test to compare with $b_n = 1/n$. Here, the point is that

$$a_n = \log\left(1 + \frac{1}{n}\right) = \frac{1}{n} \cdot \log\left(1 + \frac{1}{n}\right)^n \approx \frac{1}{n} \cdot \log e = \frac{1}{n}$$

for all large enough n, so a_n is comparable to b_n for all large enough n.

3. The answers are 18 and 25/3, respectively.