MA121 Tutorial Problems #5

1. Compute each of the following integrals:

$$\int \log(1+x^2)\,dx, \qquad \int \frac{x^3-x+1}{x+2}\,dx, \qquad \int \frac{dx}{1+e^x}, \qquad \int \frac{x^2+1}{x^3+x^2}\,dx.$$

2. Compute each of the following integrals:

$$\int \frac{2x+3}{x^2-9}\,dx, \qquad \int \frac{x+7}{x^2(x+2)}\,dx, \qquad \int \frac{2\sin x\cdot\cos x}{\sin^2 x-4}\,dx, \qquad \int x\cdot\arctan x\,dx.$$

- These are all practice problems, not a homework assignment.
- However, part of your next homework assignment will be based on these problems.
- In case you get stuck, some hints are provided on the other page of this sheet.

Hints and comments

1a. First, integrate by parts to get rid of the logarithm. You will find that

$$\int \log(1+x^2)\,dx = \int x' \log(1+x^2)\,dx = x\log(1+x^2) - \int \frac{2x^2}{1+x^2}\,dx.$$

To compute the rightmost integral, you will need to use division of polynomials.

1b. Use division of polynomials to show that

$$\frac{x^3-x+1}{x+2} = x^2 - 2x + 3 - \frac{5}{x+2}\,.$$

1c. First of all, use the substitution $u = e^x$ to show that

$$\int \frac{dx}{1+e^x} = \int \frac{e^x\,dx}{e^x(1+e^x)} = \int \frac{du}{u(1+u)}\,.$$

You can now compute the rightmost integral using partial fractions.

1d. Note that the denominator factors and then use partial fractions, namely

$$\frac{x^2+1}{x^3+x^2} = \frac{x^2+1}{x^2(x+1)} = \frac{Ax+B}{x^2} + \frac{C}{x+1}$$

for some constants A, B, C that need to be determined.

2a. Note that the denominator factors and then use partial fractions, namely

$$\frac{2x+3}{x^2-9} = \frac{2x+3}{(x-3)(x+3)} = \frac{A}{x-3} + \frac{B}{x+3}$$

for some constants A, B that need to be determined.

2b. In this case, the partial fractions decomposition is of the form

$$\frac{x+7}{x^2(x+2)} = \frac{Ax+B}{x^2} + \frac{C}{x+2}\,.$$

2c. The easiest way to do this is to set $u = \sin^2 x - 4$, in which case

$$\int \frac{2\sin x\cos x}{\sin^2 x - 4}\,dx = \int \frac{du}{u} = \log|u| + C.$$

Also, note that u is negative here because $u = \sin^2 x - 4 \leq 1 - 4 = -3$.

2d. First, integrate by parts to get rid of the arctan. You will find that

$$\int x\arctan x\,dx = \int \left(\frac{x^2}{2}\right)' \arctan x\,dx = \frac{x^2\arctan x}{2} - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx.$$

You have already computed the rightmost integral; see question (1a) above.