UNIVERSITY OF DUBLIN

XMA1213

TRINITY COLLEGE

Faculty of Science

school of mathematics

JF Maths, JF TP
JF TSM, SF TSM

Hilary Term 2008

Course 121

Monday, March 10 Regent House 14:00 – 17:00

Dr. P. Karageorgis

Attempt all questions. All questions are weighted equally.
You may use non-programmable calculators, but you may not use log tables.

1. Compute each of the following integrals:

$$\int \frac{3x+5}{x^3-x}\,dx, \qquad \int x\cos x\,dx.$$

2. Let f be defined by $f(x) = \left\{ \begin{array}{ll} 1 & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{array} \right\}$. Show that f is integrable on $[0,1]$.

3. Define a sequence $\{a_n\}$ by setting $a_1 = 4$ and

$$a_{n+1} = \frac{1}{5-a_n} \quad \text{for each } n \geq 1.$$

Show that $0 \leq a_{n+1} \leq a_n \leq 4$ for each $n \geq 1$, use this fact to conclude that the sequence converges and then find its limit.

4. Compute each of the following limits:

$$\lim_{x\to 0} \frac{e^x - x - 1}{x^2}, \qquad \lim_{x\to\infty} x\sin(1/x).$$

5. Test each of the following series for convergence:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\,n^2}{1+n^4}, \qquad \sum_{n=1}^{\infty} \left(\frac{2n}{1+3n}\right)^n.$$

6. Find the radius of convergence of the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} \cdot x^n.$$

7. Suppose f is a differentiable function such that $f'(x) = 2x \cdot f(x)$ for all $x \in \mathbb{R}$. Show that there exists some constant C such that $f(x) = Ce^{x^2}$ for all $x \in \mathbb{R}$.

8. Suppose that f is a function with

$$|f(x) - f(y)| \leq |x-y|^2 \quad \text{for all } x, y \in \mathbb{R}.$$

Using the limit definition of the derivative, show that f is actually constant.