UNIVERSITY OF DUBLIN

XMA1212

TRINITY COLLEGE

Faculty of Science

school of mathematics

JF Maths, JF TP
JF TSM, SF TSM

Michaelmas Term 2007

Course 121

Monday, December 10 Luce Hall 14:00 – 17:00

Dr. P. Karageorgis

Attempt all questions. All questions are weighted equally.
You may use non-programmable calculators, but you may not use log tables.

1. Make a table listing the min, inf, max and sup of each of the following sets; write DNE for all quantities which fail to exist. You need not justify any of your answers.

(a) $A = \{n \in \mathbb{N} : n - 1 \in \mathbb{N}\}$

(b) $B = \{x \in \mathbb{R} : 2x \leq 5\}$

(c) $C = \{x \in \mathbb{R} : |x| < y \text{ for all } y > 0\}$

(d) $D = \{x \in \mathbb{R} : |x + 1| < 1\}$

2. Let f be the function defined by

$$f(x) = \left\{ \begin{array}{cl} \frac{4x^3-7x-3}{2x-3} & \text{if } x \neq 3/2 \\ 10 & \text{if } x = 3/2 \end{array} \right\}.$$

Show that f is continuous at $y = 3/2$. As a hint, one may avoid the ε-δ definition here.

3. Show that there exists some $0 < x < 1$ such that $(x^2 - 2x + 3)^3 = (2x^2 - x + 1)^4$.

4. Find the maximum value of $f(x) = x(7 - x^2)^3$ over the closed interval $[-1, 3]$.

5. Suppose that f is a differentiable function such that

$$f'(x) = \frac{1}{1 + x^2} \quad \text{for all } x \in \mathbb{R}.$$

Show that $f(x) + f(1/x) = 2f(1)$ for all $x > 0$.

6. Let f be the function defined by

$$f(x) = \left\{ \begin{array}{cl} 2 - 3x & \text{if } x \leq 2 \\ 4 - 5x & \text{if } x > 2 \end{array} \right\}.$$

Show that f is discontinuous at $y = 2$.

7. Let A be a nonempty subset of $\mathbb{R}$ that has an upper bound and let $\varepsilon > 0$ be given. Show that there exists some element $a \in A$ such that $\sup A - \varepsilon < a \leq \sup A$.

8. Show that the polynomial $f(x) = x^4 - 2x^3 + x^2 - 1$ has exactly one root in $(1, 2)$.