MA121, Homework #2

due Monday, Nov. 12 in class

- 1. Make a table listing the min, inf, max and sup of each of the following sets; write DNE for all quantities which fail to exist. You need not justify any of your answers.
 - (a) $A = \left\{ n \in \mathbb{N} : \frac{n}{n+1} > \frac{3}{4} \right\}$ (b) $B = \left\{ x \in \mathbb{R} : x > 1 \text{ and } 2x \le 5 \right\}$ (c) $C = \left\{ x \in \mathbb{Z} : x > 1 \text{ and } 2x \le 5 \right\}$ (d) $D = \left\{ x \in \mathbb{R} : x < y \text{ for all } y > 0 \right\}$ (e) $E = \left\{ x \in \mathbb{R} : 1 \le |x - 2| < 3 \right\}$
- **2.** Let $x \in \mathbb{R}$ be such that x > -1. Show that $(1+x)^n \ge 1 + nx$ for all $n \in \mathbb{N}$.
- **3.** Let A, B be nonempty subsets of \mathbb{R} such that $\inf A < \inf B$. Show that there exists an element $a \in A$ which is a lower bound of B.
- 4. Evaluate the limit

$$L = \lim_{x \to 1} \frac{6x^3 - 13x^2 + 4x + 3}{x - 1}.$$

- **5.** Let f be a function such that $|f(x) 3| \le 5|x|$ for all $x \in \mathbb{R}$. Show that $\lim_{x \to 0} f(x) = 3$.
- **6.** Show that the function f defined by

$$f(x) = \left\{ \begin{array}{ll} 3x - 2 & \text{if } x \le 2\\ 4x - 4 & \text{if } x > 2 \end{array} \right\}$$

is continuous at y = 2.

- You are going to work on these problems during your Friday tutorials.
- When writing up solutions, write legibly and coherently. Use words, not just symbols.
- Write both your name and your tutor's name on the first page of your homework.
- Your tutor's name is Derek, if you are a TP student; otherwise, it is Pete.
- Your solutions may use any of the axioms/results stated in class (but nothing else).