UNIVERSITY OF DUBLIN

XMA1211

TRINITY COLLEGE

FACULTY OF ENGINEERING, MATHEMATICS AND SCIENCE

SCHOOL OF MATHEMATICS

JF Mathematics
JF Theoretical Physics
JF TSM, SF TSM

Trinity Term 2008

COURSE 121

Tuesday, May 27 Goldsmith Hall 9:30–12:30

Dr. P. Karageorgis

Attempt all questions. All questions are weighted equally.
Non-programmable calculators are permitted for this examination.
Log tables are available from the invigilators, if required.

1. Suppose that A is a nonempty subset of $\mathbb{R}$ that has a lower bound and let $\varepsilon > 0$ be given. Show that there exists an element $a \in A$ such that $\inf A \leq a < \inf A + \varepsilon$.

2. Show that the polynomial $f(x) = x^3 - 7x^2 - 5x + 1$ has exactly one root in $[0, 2]$.

3. Find the maximum value of $f(x) = \frac{x+1}{x^2+8}$ over the closed interval $[0, 3]$.

4. Compute each of the following integrals:
$$\int \frac{6x+9}{x^3+3x^2}\,dx, \qquad \int 2x^3 e^{x^2}\,dx.$$

5. Suppose f is continuous on $[a, b]$. Show that there exists some $c \in (a, b)$ such that
$$\int_a^b f(t)\,dt = (b-a)\cdot f(c).$$
As a hint, apply the mean value theorem to the function $F(x) = \int_a^x f(t)\,dt$.

6. Test each of the following series for convergence:
$$\sum_{n=1}^{\infty} \frac{n^2+2}{n^3+n}, \qquad \sum_{n=1}^{\infty} \frac{n!}{n^n}.$$

7. Let f be the function defined by
$$f(x) = \left\{ \begin{array}{ll} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{array} \right\}.$$
Show that f is not integrable on any closed interval $[a, b]$.

8. Suppose that $z = z(r, s, t)$, where $r = u - v$, $s = v - w$ and $t = w - u$. Assuming that all partial derivatives exist, show that $z_u + z_v + z_w = 0$.

9. Classify the critical points of the function defined by $f(x, y) = 3xy - x^3 - y^3$.

10. Compute the double integral
$$\int_0^1 \int_y^1 e^{x^2}\,dx\,dy.$$