UNIVERSITY OF DUBLIN

TRINITY COLLEGE

FACULTY OF SCIENCE

SCHOOL OF MATHEMATICS

JF Maths, JF TP   **Michaelmas Term 2007**
JF TSM, SF TSM

COURSE 121

Monday, December 10   Luce Hall   14:00 – 17:00

Dr. P. Karageorgis

Attempt all questions. All questions are weighted equally.
You may use non-programmable calculators, but you may not use log tables.

1. Make a table listing the min, inf, max and sup of each of the following sets; write DNE for all quantities which fail to exist. You need not justify any of your answers.

(a) $A = \left\{n \in \mathbb{N} : \frac{n}{2} \in \mathbb{N}\right\}$

(c) $C = \{x \in \mathbb{R} : x < y \text{ for all } y > 0\}$

(b) $B = \{x \in \mathbb{R} : 2x > 3\}$

(d) $D = \{x \in \mathbb{R} : 4x^2 \leq 4x - 1\}$

2. Let f be the function defined by

$$f(x) = \begin{cases} \frac{4x^3 - 7x + 3}{2x - 1} & \text{if } x \neq 1/2 \\ -2 & \text{if } x = 1/2 \end{cases}.$$

Show that f is continuous at $y = 1/2$. As a hint, one may avoid the ε-δ definition here.

3. Show that the polynomial $f(x) = x^4 - 2x^3 + x^2 - 1$ has exactly one root in $(1, 2)$.

4. Find the maximum value of $f(x) = (2x - 5)^2(5 - x)^3$ over the closed interval $[2, 5]$.

5. Let f be the function defined by

$$f(x) = \begin{cases} 2 - 2x & \text{if } x < 1 \\ 4 - 5x & \text{if } x \geq 1 \end{cases}.$$

Show that f is discontinuous at $y = 1$.

6. Let $x \in \mathbb{R}$ be a real number such that $2 - nx \geq 0$ for all $n \in \mathbb{N}$. Show that $x \leq 0$.

7. Show that $3x^4 + 4x^3 \geq 12x^2 - 32$ for all $x \in \mathbb{R}$.

8. Show that the set $A = \left\{\frac{n+1}{n} : n \in \mathbb{N}\right\}$ is such that $\inf A = 1$.

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