UNIVERSITY OF DUBLIN

XMA1211

TRINITY COLLEGE

FACULTY OF SCIENCE

SCHOOL OF MATHEMATICS

JF Mathematics **Trinity Term 2007**
JF Theoretical Physics
JF TSM, SF TSM

COURSE 121

Tuesday, May 22 RDS 9:30–12:30

Dr. P. Karageorgis

Attempt all questions. All questions are weighted equally.

1. Suppose that A is a nonempty subset of $\mathbb{R}$ that has an upper bound, and let B be the set of all upper bounds of A. Show that $\inf B = \sup A$.

2. Let $a \in \mathbb{R}$ be a given number and let f be the function defined by

$$f(x) = \left\{ \begin{array}{ll} ax^2 + 2x & \text{if } x \neq 2 \\ 2a + 8 & \text{if } x = 2 \end{array} \right\}.$$

Find the value of a for which f is continuous at $y = 2$.

3. Find the minimum value of $f(x) = x^4 + 4x^3 - 8x^2 + 2$ over the whole real line.

4. Let $a, b, c \in \mathbb{R}$ be some fixed constants such that $\frac{a}{3} + \frac{b}{2} + c = 0$. Show that

$$ax^2 + bx + c = 0 \quad \text{for some } x \in (0, 1).$$

As a hint, apply the mean value theorem to a function whose derivative is $ax^2 + bx + c$.

5. Suppose that f is a function which satisfies the inequality

$$|f(x) - f(y)| \leq |x - y|^2 \quad \text{for all } \ x, y \in \mathbb{R}.$$

Show that f is actually constant.

6. Evaluate each of the following integrals:

$$\int \frac{4x^2 - 15x + 12}{x^3 - 5x^2 + 6x}\, dx, \qquad \int \frac{x^3 - x + 1}{x + 1}\, dx.$$

As a hint for the first integral, you might want to factor the denominator.

7. Test each of the following series for convergence:

$$\sum_{n=0}^{\infty} \frac{n!}{(2n)!}, \qquad \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}, \qquad \sum_{n=1}^{\infty} \frac{n^2 + 2}{n^3 + n}.$$

8. Evaluate each of the following sums:

$$\sum_{n=0}^{\infty} \frac{2^{n+1}}{3^{n+2}}, \qquad \sum_{n=2}^{\infty} \frac{e^n}{n!}, \qquad \sum_{n=1}^{\infty} \frac{(-1)^n\, 9^{n+1}}{(2n)!}.$$

9. Let f be the function defined by

$$f(x) = \left\{ \begin{array}{ll} 1 & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{array} \right\}.$$

Show that f is integrable on $[0, 1]$.

10. Define a sequence $\{a_n\}$ by setting $a_1 = 2$ and

$$a_{n+1} = \frac{1}{3 - a_n} \quad \text{for each } n \geq 1.$$

Show that $0 < a_{n+1} \leq a_n \leq 2$ for each $n \geq 1$. Use this fact to conclude that the sequence converges and then find its limit.