UNIVERSITY OF DUBLIN XMA1212

TRINITY COLLEGE

JF Maths   **Michaelmas Term 2006**
JF TP
JF TSM

COURSE 121

Monday, December 11   Goldsmith Hall   14:00 – 17:00

Dr. P. Karageorgis

Attempt all questions. All questions are weighted equally.

1. Make a table listing the min, inf, max and sup of each of the following sets; write DNE for all quantities which fail to exist. You need not justify any of your answers.

(a) $A = \left\{ n \in \mathbb{N} : \frac{1}{n} < \frac{2}{3} \right\}$

(c) $C = \{ x \in \mathbb{Z} : x \geq 3 \text{ and } 2x < 7 \}$

(b) $B = \{ x \in \mathbb{R} : x^2 < -1 \}$

(d) $D = \{ x \in \mathbb{R} : |x + 1| < 1 \}$

2. Show that $2^n \geq n + 1$ for all $n \in \mathbb{N}$.

3. Show that there exists some $0 < x < 1$ such that $2x^2 + 3x^3 = x^5 + 1$.

4. Let f be a function such that $|f(x) - 1| \leq 2\,|x|$ for all $x \in \mathbb{R}$. Show that $\lim_{x \to 0} f(x) = 1$.

5. Let A, B be nonempty subsets of $\mathbb{R}$ such that $\sup A \leq b$ for all $b \in B$. Show that

$$\inf A \leq \inf B.$$

As a hint, you might wish to show that $\inf A \leq \sup A$ and that $\sup A \leq \inf B$, instead.

6. Let f be the function defined by

$$f(x) = \begin{cases} \frac{x^3 + x^2 - 2}{x - 1} & \text{if } x \neq 1 \\ 5 & \text{if } x = 1 \end{cases}.$$

Show that f is continuous at all points. As a hint, one may avoid the ε-δ definition here.

7. Given the set $A = \left\{ \frac{x}{x^2 + 1} : x \in \mathbb{R} \right\}$, show that $\inf A = -\frac{1}{2}$.

8. Show that the function f defined by

$$f(x) = \begin{cases} 2x & \text{if } x \leq 1 \\ x + 3 & \text{if } x > 1 \end{cases}$$

is discontinuous at $y = 1$.