MA2E01 Tutorial solutions #2

1. Find the equation of the plane that passes through the points

$$A(1,2,4), \qquad B(1,-1,0), \qquad C(2,3,1).$$

The normal vector to the plane is given by the cross product

$$\overrightarrow{AB} \times \overrightarrow{AC} = \langle 0,-3,-4\rangle \times \langle 1,1,-3\rangle = \langle 13,-4,3\rangle\,.$$

Since the plane passes through the point $A(1,2,4)$, its equation is then

$$13(x-1) - 4(y-2) + 3(z-4) = 0 \quad\Longrightarrow\quad 13x - 4y + 3z = 17.$$

2. Find the equation of the line through $(1,2,3)$ which is perpendicular to the line

$$x = 1+t, \qquad y = 2-3t, \qquad z = 2t$$

and also parallel to the plane $2x - z = 1$. *Hint: if a line is parallel to a plane, then it is perpendicular to the normal vector of the plane.*

The desired line is perpendicular to both $\langle 1,-3,2\rangle$ and $\langle 2,0,-1\rangle$, so its direction is

$$\boldsymbol{n} = \langle 1,-3,2\rangle \times \langle 2,0,-1\rangle = \langle 3,5,6\rangle\,.$$

Since it passes through $(1,2,3)$ with direction $\boldsymbol{n}$, its equation is then

$$x = 1+3t, \qquad y = 2+5t, \qquad z = 3+6t.$$

3. Sketch the level curves $f(x,y) = k$ in the case that $f(x,y) = x + y^2$ and $k = 0,1,2$. Use these level curves to draw a rough sketch of the graph of f.

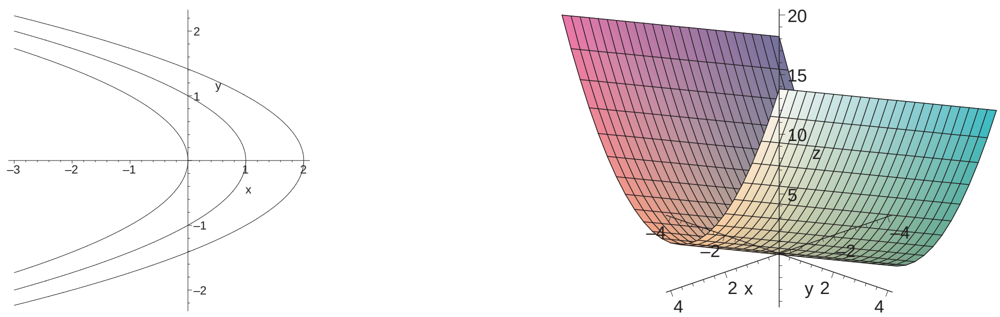

Figure 1: The level curves $x + y^2 = k$ and the graph of $z = x + y^2$.

4. Compute the partial derivatives f_x, f_y and f_{xy} in the case that $f(x, y) = y^2 e^{xy}$.

In this case, $f_x(x, y) = y^2 e^{xy}(xy)_x = y^3 e^{xy}$ and the product rule gives

$$f_y(x, y) = 2ye^{xy} + y^2 e^{xy}(xy)_y = 2ye^{xy} + xy^2 e^{xy}.$$

Differentiating f_x with respect to y, one also finds that

$$f_{xy}(x, y) = 3y^2 e^{xy} + y^3 e^{xy}(xy)_y = 3y^2 e^{xy} + xy^3 e^{xy}.$$