## MA2E01 Tutorial problems #5

(due at the end of your tutorial)

1. Use polar coordinates to evaluate the integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \cos(x^2 + y^2) \, dy \, dx.$$

2. Use polar coordinates to evaluate the integral

$$\int_0^2 \int_y^{\sqrt{8-y^2}} \frac{dx \, dy}{\sqrt{1+x^2+y^2}}.$$

**3.** The sphere of radius *a* around the origin is given by the parametric equation

$$\boldsymbol{r}(\theta,\phi) = \langle a\sin\phi\cos\theta, a\sin\phi\sin\theta, a\cos\phi\rangle,$$

where  $0 \le \theta \le 2\pi$  and  $0 \le \phi \le \pi$ . Use this fact to compute its area.

4. A lamina with density  $\delta(x, y) = x + y$  is bounded by the x-axis, the line x = 1 and the curve  $y = \sqrt{x}$ . Find its mass and also its center of gravity.